RESEARCHES

ON THE

PERFORMANCE OF THE SCREW PROPELLER

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W. F. DURAND,

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RESEARCHES ON THE PERFORMANCE OF THE SCREW PROPELLER.

INTRODUCTORY.

During the summer of 1897 certain tests were carried out on a series of propellers, all of the same pitch ratio and form of blade, but with varying widths, giving an area regularly varying between .18 and .72. These experiments were carried out by mounting the propellers on a special frame extending over the bow of a boat used as a carrier, and propelled by its own propeller at the stern. By running over a carefully measured course of 1,000 feet and by suitable measurements of time, distance, revolutions of model propeller, thrust developed, and turning moment required, it became possible to determine all quantities entering into the performance of the propeller and to completely determine and analyze the same. These experiments have been elsewhere reported * and are simply mentioned here as a starting-point for those under present report.

Further work with similar propellers of varying pitch ratios was delayed by various causes, but at length this longer program became possible under much improved conditions in the Hydraulic Laboratory at Cornell University, and later through the aid of a grant from the Carnegie Institution, under which auspices the entire work has been reviewed and carried forward to completion so far as this particular series of propellers is concerned.

In addition to the work planned under the general program, certain special and collateral experimental researches have been included in the work, all of which are noted at appropriate points in the present report. A preliminary and partial report on this work was made in the form of a paper read before the Society of Naval Architects and Marine Engineers in the fall of 1905 and from which extracts will be made in the course of the present more complete report.

The propellers upon which the experiments were made are 49 in number and are distributed over the field of pitch and area ratios as shown by table 1.

The diameter of all propellers was 12 inches and the diameter of hub 2.4 inches. The number of blades was four and the shape of all blades was elliptical, the maximum width in tenths of the radius being expressed by the area number in the table.

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* Transactions Society of Naval Architects and Marine Engineers, vol. v, p. 107.

TABLE 1.

Area No.		Pitch Ratios.						
		.9	1.1	1.3	1.5	1.7	1.9	2.1
2	Area Ratios.....	.18	.18	.18	.18	.18	.18	.18
3		.27	.27	.27	.27	.27	.27	27
4		.36	.36	.36	.36	.36	.36	.36
5		.45	45	.45	.45	.45	.45	.45
6		.54	.54	.54	.54	.54	.54	.54
7		.63	.63	.63	.63	.63	.63	.63
8		.72	.72	.72	.72	.72	.72	.72

The forms of the blade contours collectively are shown in fig. 1, and the thickness lay-out along the central axis of the blade by fig. 3, while the entire number in general view are shown in fig. 2. For making the propellers a wooden pattern was first prepared for the given pitch and for the widest blades or No. 8. This propeller being cast, the pattern was reduced in blade width

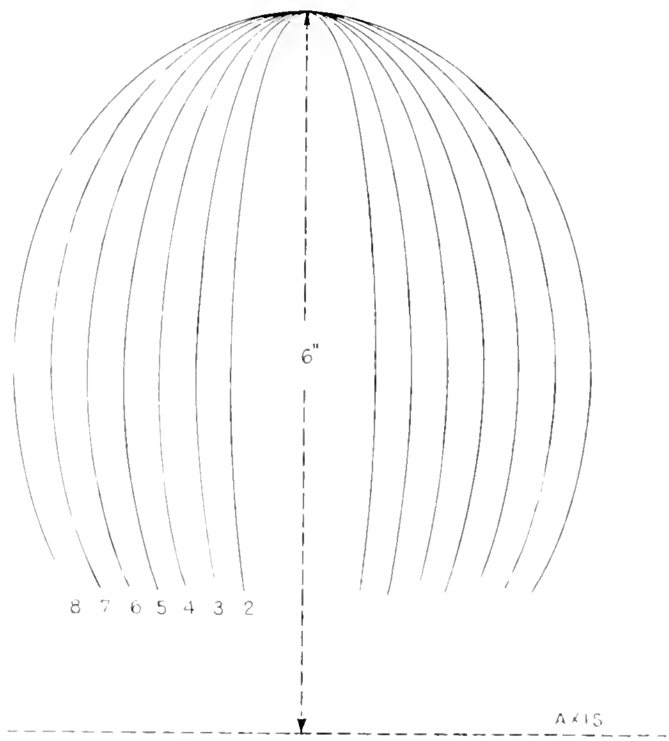

FIG. 1.—COLLECTIVE BLADE CONTOUR.

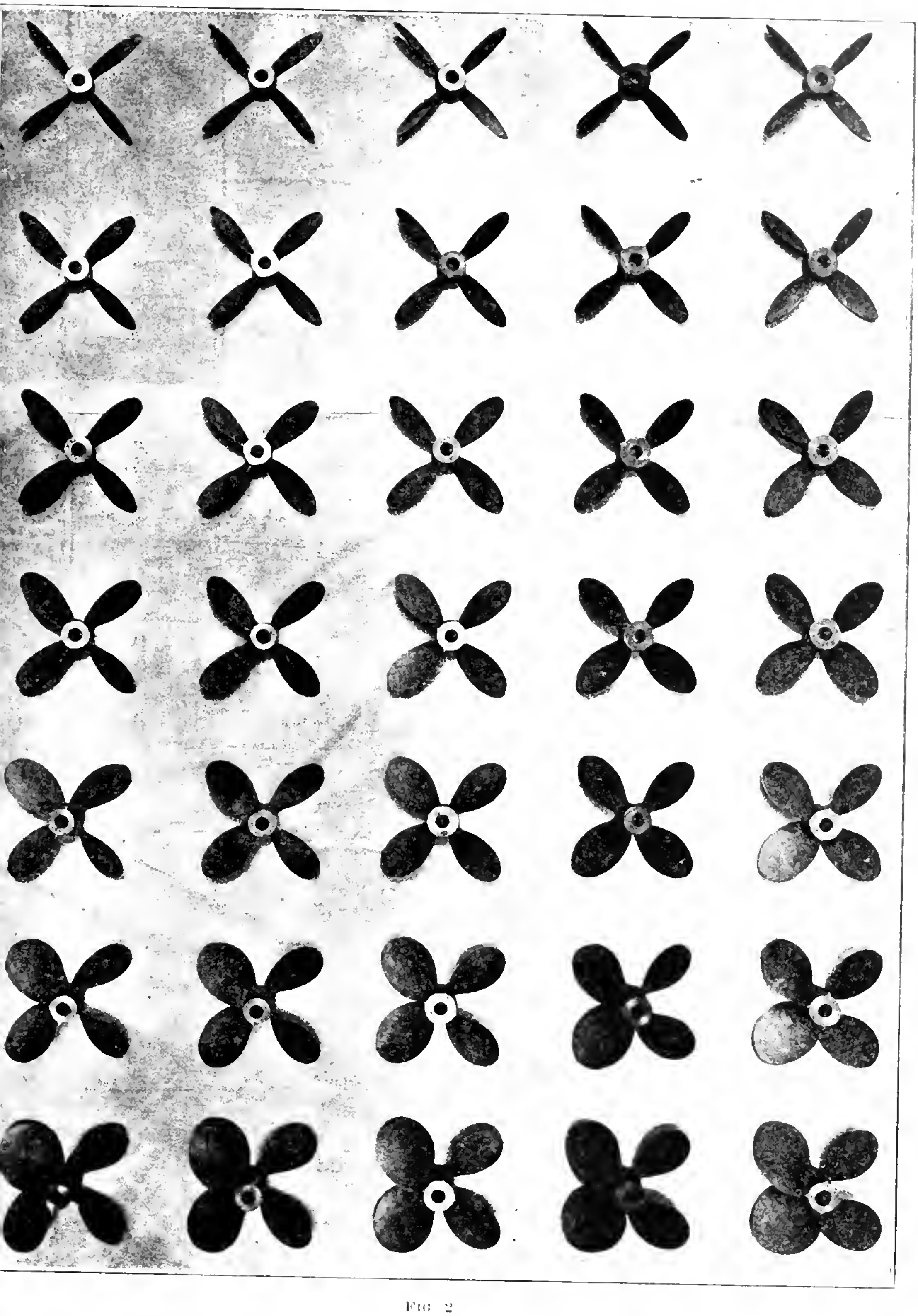

Fig. 2

and thickness to conform with the lay-out in figs. 1 and 3. No. 7 was then cast, and so throughout the series to No. 2, but one original pattern being thus required for the entire number of any one pitch ratio.

For the measurement of the pitch thus resulting, the device shown in fig. 5 * was used. The propeller was bored, fitted on a mandrel and swung in a lathe, face outward. Fitted on the same mandrel was a drum of diameter, say, .7 that of the propeller. On this drum was secured a strip of paper.

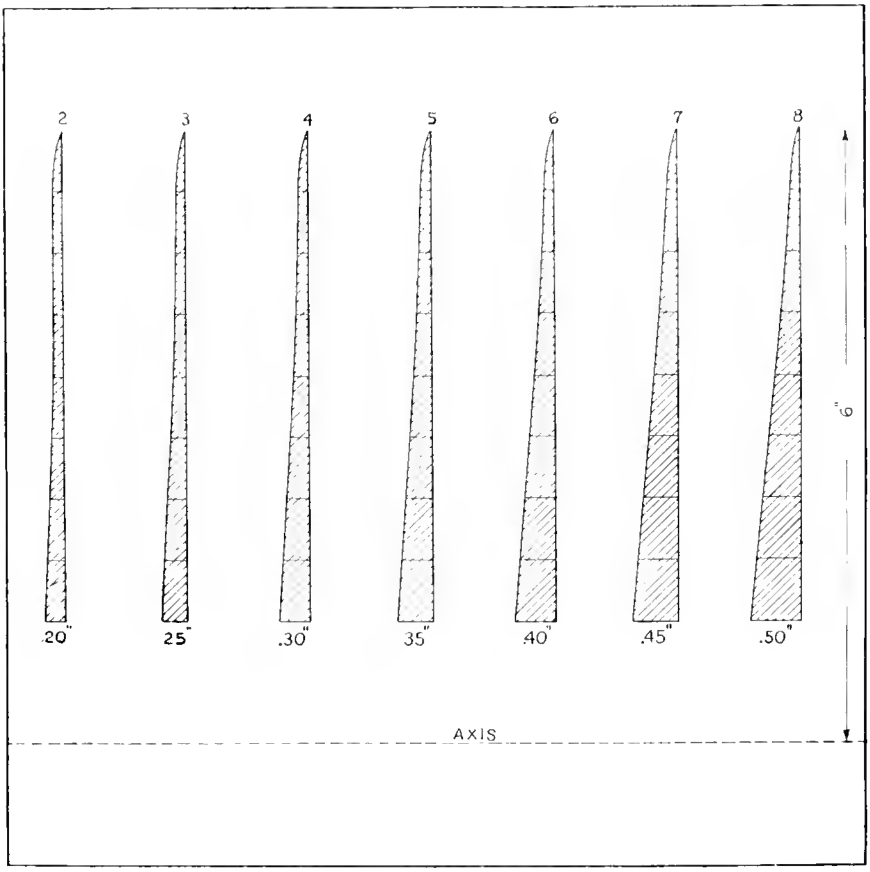

FIG. 3.

A square steel bar was then fitted in the tool post with one edge parallel to the lathe axis. At the end next the propeller the bar was drawn down to a point lying in the upper and inner edge of the bar. Farther back, and opposite the drum, a nick was made in the same edge. Now, it is obvious that if the bar be run in till the edge above referred to is just clear of the drum surface, and if the carriage be so moved longitudinally that the point of the

* Description of this device extracted from paper before the Society of Naval Architects and Marine Engineers, Transactions, volume v, page 107.

bar rests constantly on the surface, then all points on the bar will trace relative to the drum, like helices, and hence like that traced by the end of the bar on the driving face of the propeller. A pencil held in the nick of the bar will, therefore, describe on the paper a copy of the helix on the propeller, and will, therefore, serve to determine the pitch across the blade at this radius. Four such drums were used at radii approximately .3, .5, .7, and .9 of the outer radius, the hub radius being .2 the outer radius.

In this way a sufficiently complete knowledge of the history of the variation of pitch over the driving face could be determined. Conversely the same apparatus was used to guide the dressing of the propeller down to the desired

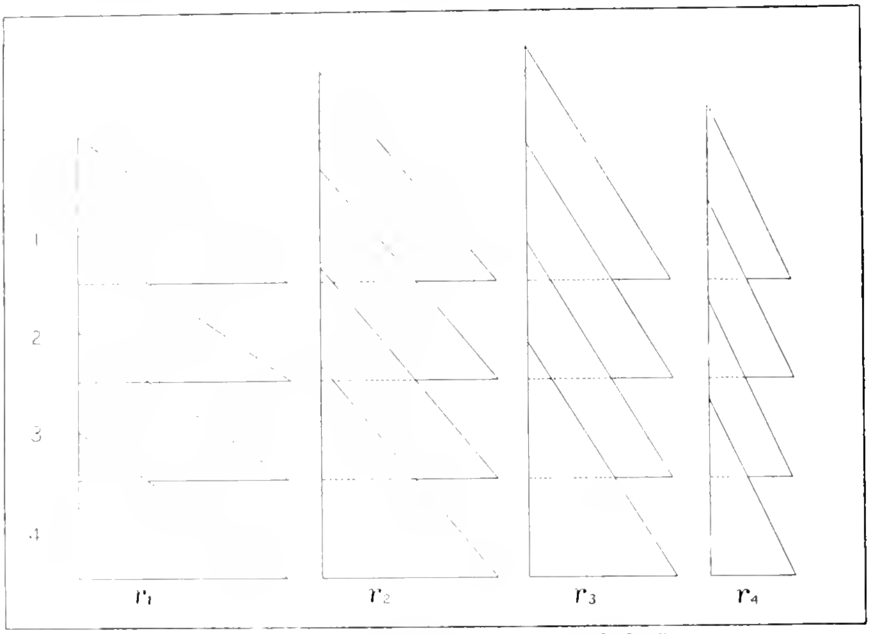

FIG. 4.—TYPICAL PITCH LINES.—PITCH 1.3 NO. 5.

degree of constancy as to pitch. To this end a line was laid off on the paper making an angle a with the transverse determined by the equation:

$$\tan a = \frac{p}{2\pi r}$$

where p is the pitch desired and r the given radius.

By comparing the path of the nick as given by the actual surface with that which should be given, the points which are too high may be easily determined. At such points a little hollow was chipped out of a depth sufficient to bring the nick to the line. A series of such spots being chipped out across the blade, it is evident that the bottoms of such points are on the surface desired. This being repeated on the other radii, similar series of points were determined. The whole surface was then reduced by grinding and filing until the bottoms of the holes were barely removed. The driving face was then taken as a sufficiently close approximation to the form desired. The blade was then trimmed

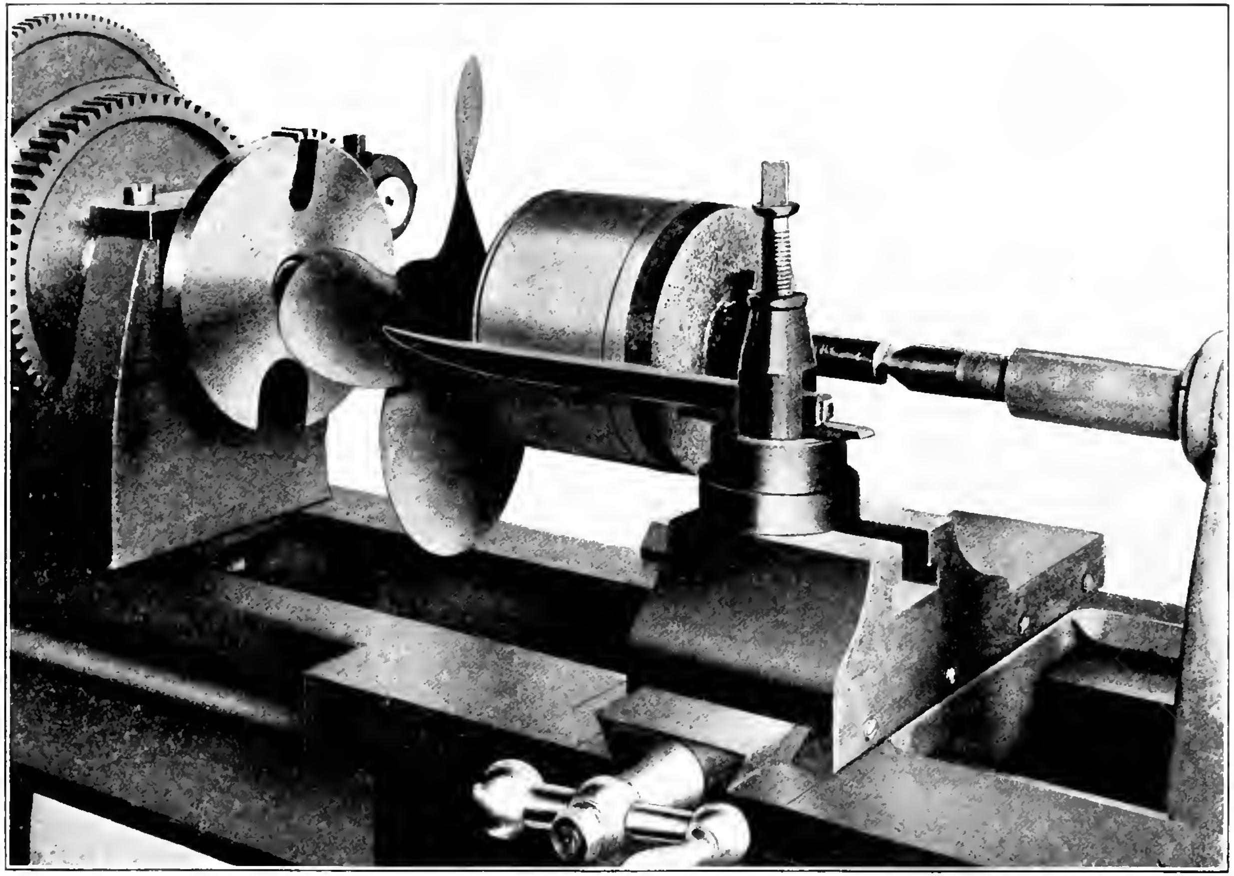

FIG. 5.—ARRANGEMENT FOR DETERMINATION OF PITCH.

to the desired contour, the original casting having been made purposely a little larger in all dimensions. Next the back was reduced until the lay-out of the thickness was that called for by the drawings, and the propeller was ready for use.

At the completion of the entire series of tests all propellers were again carefully measured for pitch by means of the same general method. The record for any one propeller consists of four oblique lines for each of the four blades and thus gives the complete record across the blades at 16 distributed points for the entire propeller. A typical chart or record for one blade is shown in fig. 4.

From these records the mean resulting pitch at each radius was computed by means of the equation $p = 2\pi r \tan \alpha$, thus giving actual values at the four selected locations on each blade and including all instrumental and operative deviations from the original. It was then assumed that the effective pitch would vary nearly as the product of the blade width by the square of the radius. A special mathematical investigation indicated that for ordinary working conditions the index for the radius should be somewhat less than 2, but the difference was of such a character as to involve no error of appreciable magnitude. On this basis the individual values of the pitch as measured were multiplied each by the product of the blade width into the square of the radius as a weight factor. These results being then summed for one propeller, the entire summation was divided by the summation of the products above-mentioned and the quotient was taken as the mean effective pitch over the face of the blades.

Values of the pitch thus obtained are given in tabular form in table 2, and naturally depart slightly from the nominal values of table 1. In the final reduction of the results the values of thrust and work for these actual values of the pitch were reduced to the corresponding values for table 1 by the application of slight corrections determined by the well-known methods of graphical representation and analysis.

TABLE 2.

Nominal pitch.	Actual pitch of propellers as tested.						
	Area No. 2.	Area No. 3.	Area No. 4.	Area No. 5.	Area No. 6.	Area No. 7.	Area No. 8.
.9	.942	.906	.930	.940	.918	.920	.914
1.1	1.123	1.106	1.113	1.118	1.115	1.120	1.075
1.3	1.271	1.329	1.307	1.308	1.309	1.306	1.307
1.5	1.542	1.531	1.505	1.513	1.511	1.511	1.519
1.7	1.713	1.740	1.690	1.696	1.713	1.721	1.712
1.9	1.874	1.871	1.875	1.903	1.870	1.886	1.873
2.1	2.086	2.072	2.075	2.067	2.141	2.107	2.098

APPARATUS AND METHODS EMPLOYED.

Before taking up the presentation and discussion of the results of the various tests, it will be of interest to describe briefly the apparatus employed for the observation of the various items required by the purposes in view.

The experiments were carried out in the canal of the hydraulic laboratory at Cornell University. This canal or tank is about 340 feet in length by 16 feet in width and 10 feet in depth. It is provided with water from the reservoir above, through double shut-off gates and an intermediate lock or measuring chamber. Once filled, the water is quiescent, and the level may be maintained where desired by an adjustable weir at the western end.

The principal items of the equipment of this canal are as follows:

(1) A carriage or truck spanning the canal and carrying the propellers to be tested and the apparatus required, and running on a track extending the length of the canal.

(2) A transmission dynamometer for the measurement of the power absorbed by the propeller.

(3) Means for driving the propeller through the dynamometer at speeds varying over a wide range, and for any run in constant proportion to the speed of the truck along the rails.

(4) A thrust dynamometer for measuring the thrust developed.

(5) Means for registering time, distance, and revolutions.

CARRIAGE OR TRUCK.

This consists essentially of a platform about 8 by 18 feet, and carried primarily on two 10-inch steel channels spanning the canal, with shorter I-beams framed in between at each end. Between the latter are carried the wheels and motive power. The latter consists of a 15 H. P. electric motor geared down by a double reduction to the driving axle. The range of speeds is from about 100 to 600 feet per minute, or from 1 to 6 knots. The speed of the motor is under control by means of a rheostat at the starting end and in circuit with the field coils of the generator in the power house. By means of this setting rheostat an adjustable E. M. F. can be readily provided at the motor, and thus any speed obtained within the limits provided. For the common range of speeds, 300 to 500 feet per minute, the distance required for acceleration is found not to exceed 30 to 50 feet.

TRANSMISSION DYNAMOMETER.

The form adopted is that used in certain experiments previously made on Cayuga Lake and elsewhere described.* This consists of a special form of rope dynamometer as shown in fig. 6. The rope coming from a sheave on the propeller shaft (see fig. 8 end view) leads around the sheaves *H* and *I*, and then around the driving sheave *G*. These sheaves *H* and *I* are mounted with

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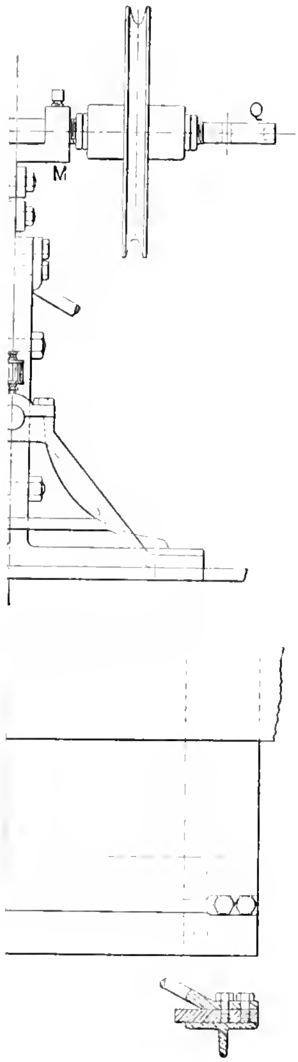
* Transactions Society of Naval Architects and Marine Engineers, vol. v, p. 107.



Q
M

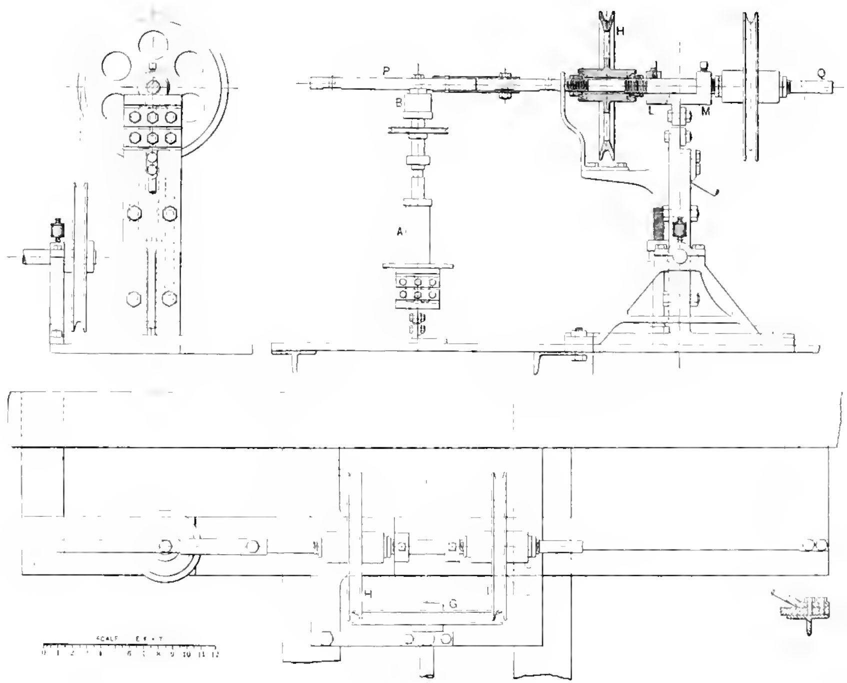


FIG. 6. TRANSMISSION DYNAMOMETER.

ball-bearings on a shaft PQ, which is carried by a block LM, the latter being connected to the base by a pair of thin plates of spring steel. This is the well-known Emery support or substitute for a knife-edge, and for slight movements is almost perfectly frictionless, at the same time affording rigidity in the directions desired. The sheaves H and I and their shaft thus form a balanced rocking system or lever pivoted in the middle and therefore without deflection so long as the tension of the rope on the two sides is the same. When running, however, the difference between the tensions on the tight and loose sides determines a moment which tends to throw this arm down. The downward thrust is then measured either by a spring dynamometer, or by a specially designed hydraulic step connected to a mercury manometer, and shown also in the figure.

A is a cylinder with plunger fitted with extra care so as to work with small leakage of oil and almost perfect absence of metallic contact. The head of the plunger rod is spherical and is connected to the rock shaft through the socket B, which provides a ball-bearing rest for the spherical head and thus absence of angular constraint between the two. The base of the cylinder A is also carried on a flexible support so that any slight angular movement which may be given to it as a result of the up-and-down motion of B will be free from all restraint. The downward thrust of the shaft PQ is thus transmitted to the plunger and thence to the oil in the lower part of the cylinder, the pressure of which is read or registered by an ordinary open mercury manometer. The movement of the shaft PQ is limited by stops to about 0.25 inch on either side of the position of equilibrium. A hand-pump is connected up for supplying oil to the cylinder, and this, together with an overflow, makes it an easy matter to keep the shaft floating between the stops during the runs.

In order to remove any residual frictional resistance to the longitudinal movement of the plunger in the cylinder, the plunger rod is provided with means for rotating it on its axis either by hand when reading, or automatically, such motion being allowed by the spherical joint at the top as referred to above. Thus, a very delicate measure of the forces is obtained, the reading being the height of the mercury column, which may reach values of 12 to 15 inches or more. The location of the arrangement relative to the shaft PQ is adjustable so that the capacity of the dynamometer may be increased with the same range of reading by lengthening the effective arm of the plunger about the axis of the rocking couple.

The indications of the dynamometer are connected with the actual forces through a calibration by means of a prony brake placed on the propeller shaft in lieu of the propeller. For this purpose the water in the canal is drawn down, leaving the brake out of water, and thus convenient for manipulation. All friction of the bearings in the dynamometers, small as it may be, is thus eliminated, and experience with this form of dynamometer shows that the indications are delicate and accurate and the calibration constant.

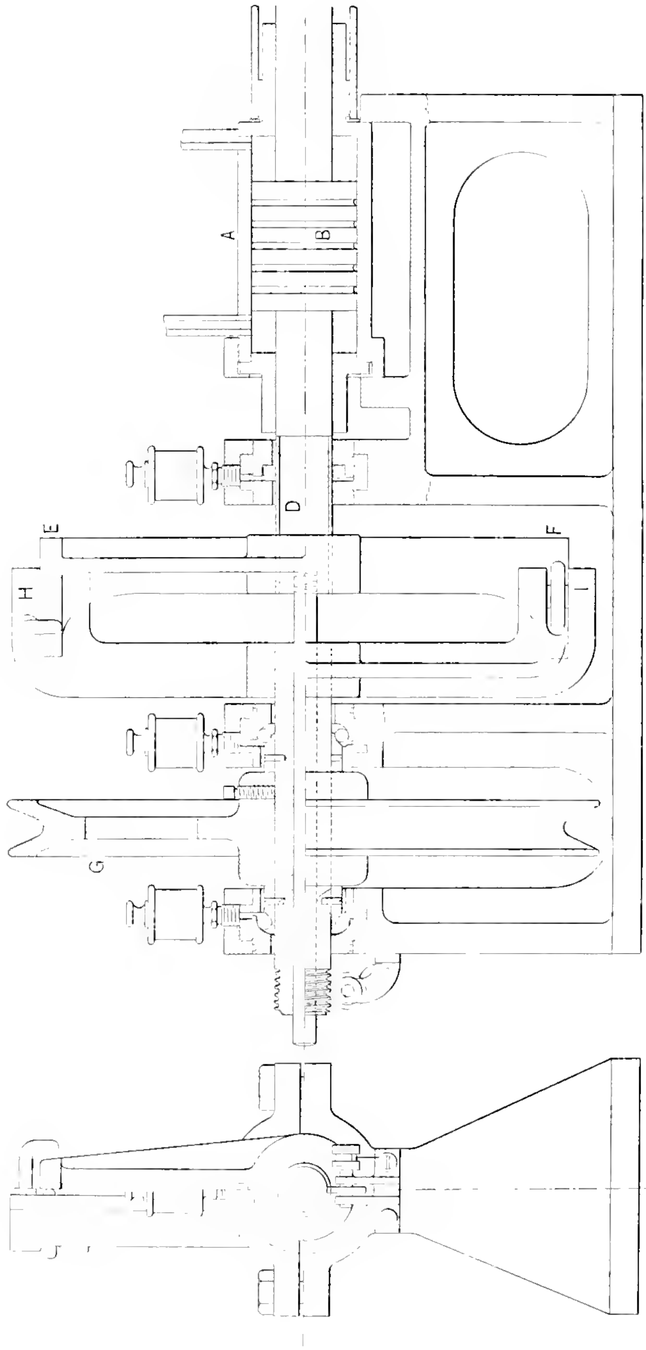
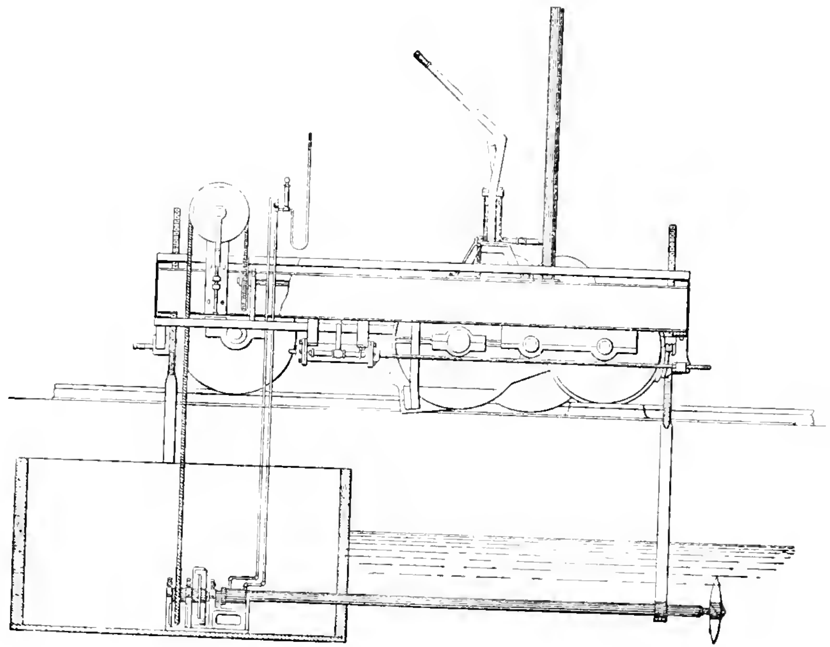


FIG. 7.—THRUST DYNAMOMETER.

Side View.



End View.

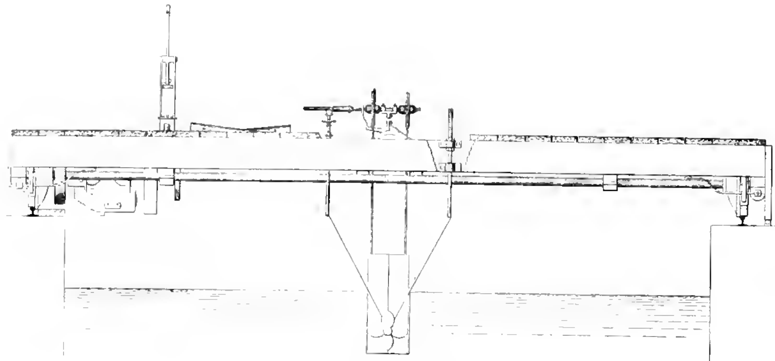
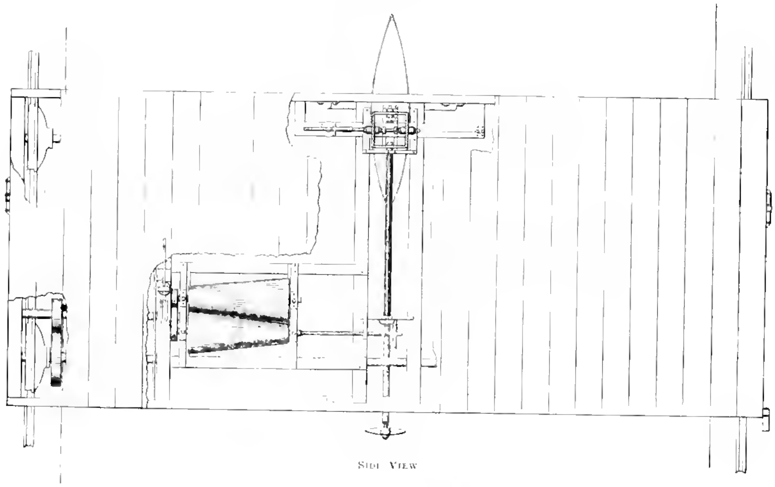
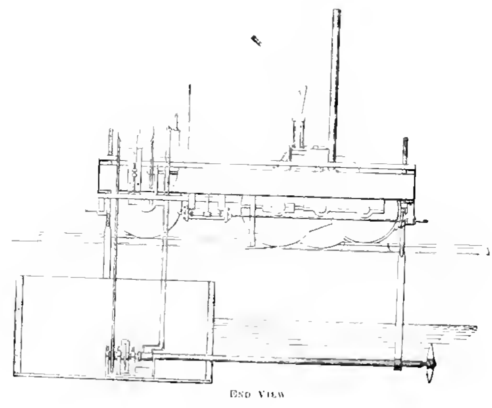


Fig. 8. General Arrangement of Apparatus



Side View



End View

THRUST DYNAMOMETER.

The thrust dynamometer is shown in fig. 7 and consists of a cylinder *A* with plunger *B*, to the rod of which is coupled the after end of the propeller shaft. The plunger rod is also continued backward as shown at *D*, and carries a pair of driving arms *EF*. The driving pulley *G* is carried on a shaft running in ball-bearings as shown. The forward part of the shaft carries a second pair of arms *HI*, engaging with those on the plunger rod. The connection between these pairs of arms is by means of a pair of small steel rollers carried by the arms *HI* and resting on *EF*. These rollers are carried also in ball-bearings, and the rollers are so adjusted that forward and backward motion of the plunger and rod is allowed by the rolling of the surfaces of one pair of arms on the rollers carried on the other pair. The torque is thus transmitted from the driving pulley to the plunger rod, with perfect freedom for the latter to move longitudinally back and forth. These various parts are carried in a small caisson or boat, pointed fore and aft, and hung from the truck by adjusting bolts.

The shaft, which is about 8 feet in length, is carried in a pipe to a forward support hung also from the truck. Here the pipe carries a ball-bearing on which the shaft rotates and moves longitudinally as may be required. The other bearings are distributed as shown and the cylinder *A* is simply supported between the two vertical faces of the base plate. The plunger and rod with propeller shaft attached is thus free to rotate and to move back and forth without constraint from its surroundings.

In operation the cylinder forward of the plunger is filled with oil connected to an open mercury manometer and the forward pull developed by the propeller is thus opposed by the pressure of the oil. The manometer thus becomes a delicate means for measuring the force as desired. The space behind the plunger is also filled with oil under pressure from a standpipe with a small reservoir on the upper end, and at the height of the manometer. This balances the column of oil in the pipe between the manometer and the cylinder, and leaves the mercury in the former free to record simply the forces developed by the propeller. A small hand-pump is provided for forcing oil in forward of the plunger, as well as an overflow for allowing it to escape, so that between the two the location of the plunger may be adjusted as desired and leakage of oil may be made up as necessary. The motion of rotation eliminates practically all frictional resistance to the longitudinal motion of the shaft, so that all of the pull developed by the propeller is transmitted to the oil, and the apparatus as a whole is found to provide an unusually delicate and accurate measure of the actual forces in play.

The automatic registration of the forces in both the power and thrust dynamometers is made by means of a special type of open-end mercury manometer, as already referred to above. The indications of the mercury column are registered by the use of an aluminum float carrying a slender steel rod moving in guides, and to which is attached a light pen carriage and pen. The

latter is held by a very light spring against the paper, and the whole is free to rise and fall in answer to the movement of the column of mercury. In practice it is found that this form of recording device is reliable and sensitive, and that it can be depended upon to respond with certainty to the movements of the mercury column. The vertical drum carrying the paper is given motion from the driving axle, so reduced as to revolve once for a complete run over the course. The indications of the manometers are thus recorded on a distance abscissa, and show the history of the variation of the forces throughout the entire course of the run.

VARIABLE SPEED CONTROL.

This is effected by driving the propellers through the transmission dynamometer by a shaft connected through an Evans friction cone and appropriate

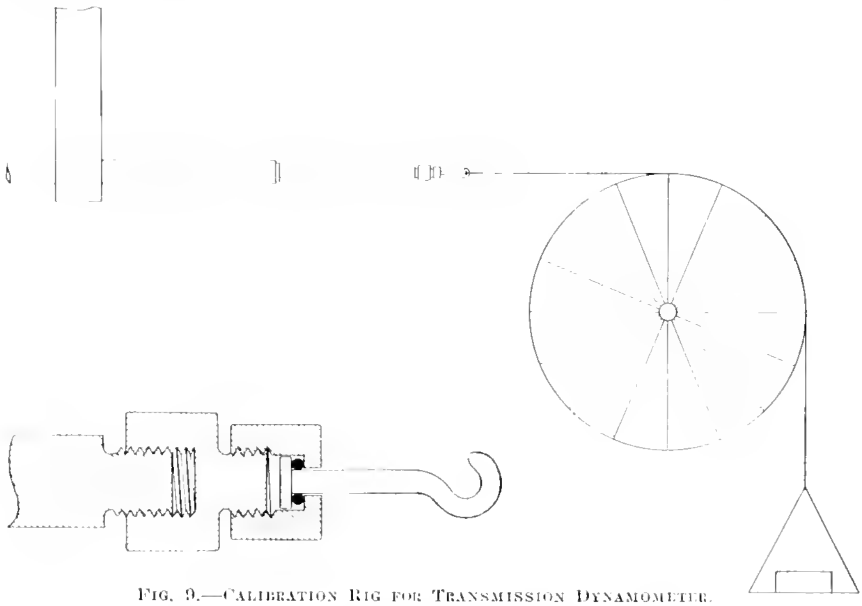


FIG. 9.—CALIBRATION RIG FOR TRANSMISSION DYNAMOMETER.

gearing to the motor. By means of a single change of gears and the friction cone, a range of revolutions from about 120 to 700 at a speed of about 300 feet per minute is thus provided for.

RECORDS OF TIME, DISTANCE, AND REVOLUTIONS.

These records are all made by a chronograph of special form, and recorded on a strip of moving paper. The time signals are furnished by a clock with electric seconds break attachment. The distance marks are given by an electric contact arrangement along the rails, and the revolutions by an electric contact attachment counting every fifth revolution of the propeller shaft. In figs. 8 and 10 are shown a general arrangement plan of the truck and apparatus with a general view of the same from the rear and looking forward.

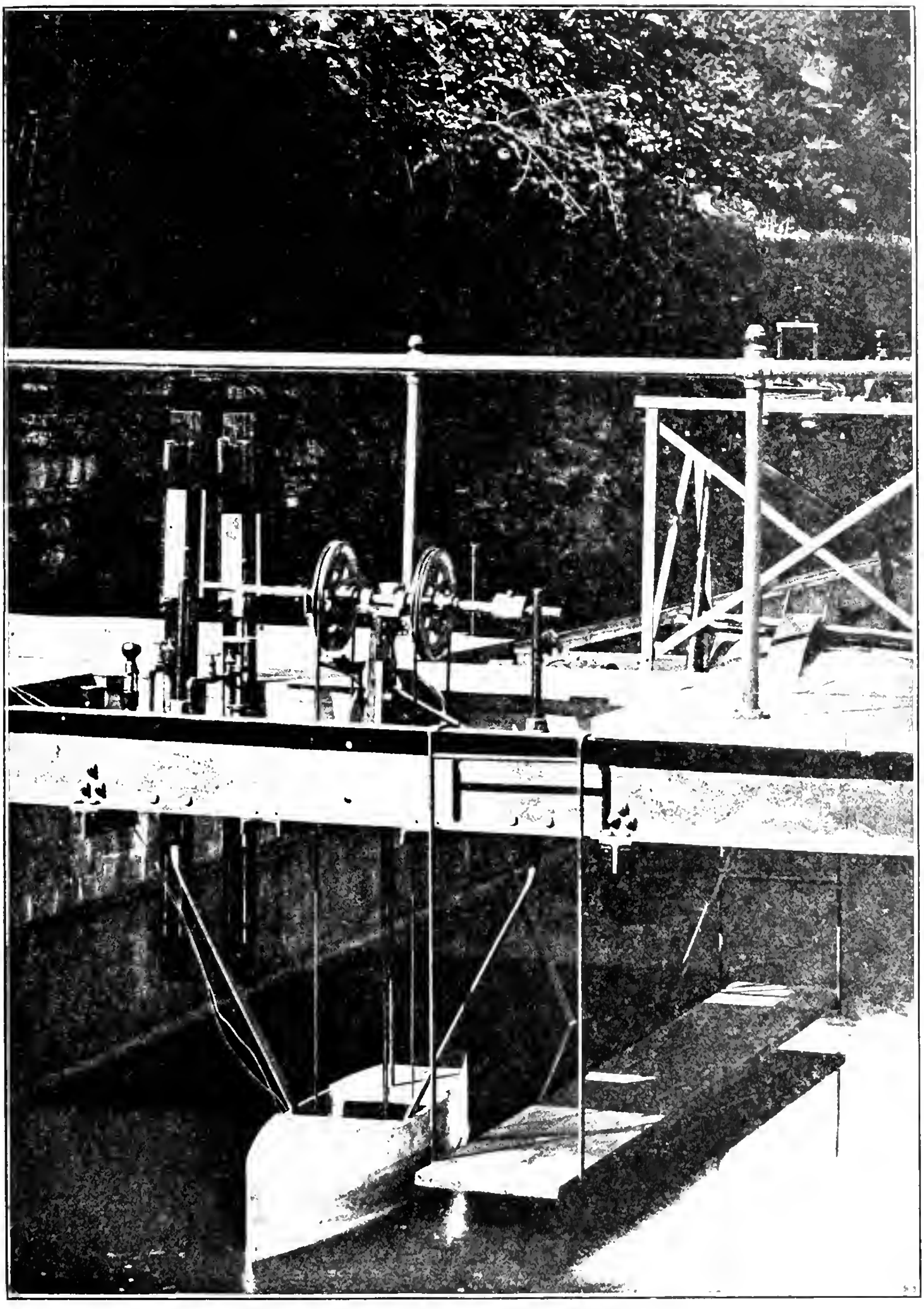


FIG. 10

CALIBRATION.

For the purpose of connecting the various necessary devices of the equipment with actual quantities it was necessary that careful calibrations should be made of the registration of the thrust and transmission dynamometers.

The thrust calibration was obtained by means of a direct pull upon the end of the shaft. For this purpose there was mounted upon the shaft a small rod with journal set with ball-bearing, in order that the cord extending from its end should receive as little torque as possible, and in practice it was found that almost no constraint was necessary to prevent the rotation of this member. A bicycle wheel rim was then mounted in a vertical plane containing the axis

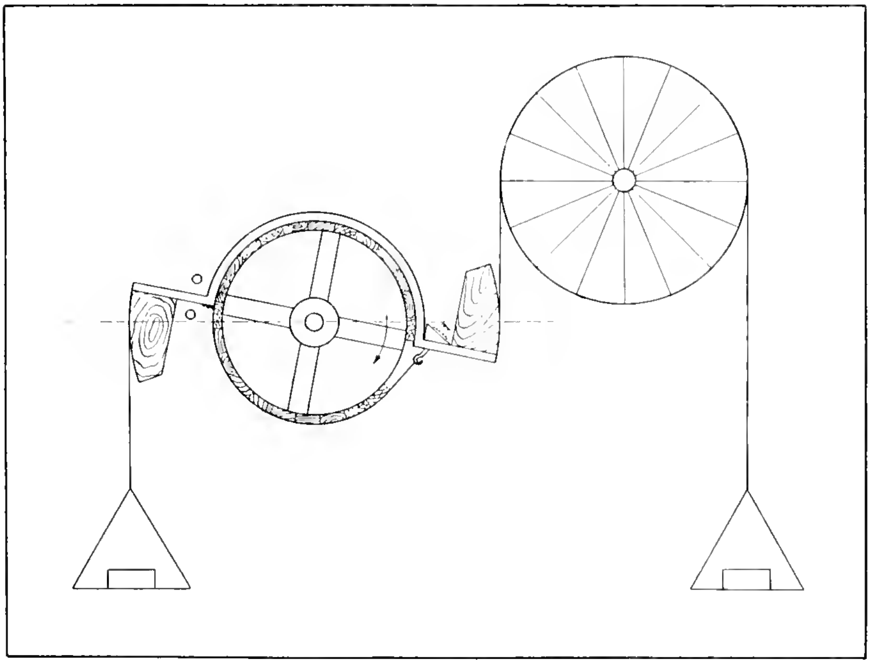


FIG. 11.—BALANCED PRONY BRAKE FOR CALIBRATION OF TRANSMISSION DYNAMOMETER.

of the shaft, and with its upper edge in direct line with that axis. A heavy fish-line was used for attaching a scale of weights to the rod above-mentioned through the intermediary of the bicycle wheel, over which it passed. This general arrangement is indicated in fig. 9. The bicycle wheel being mounted for trial with a load of 2 or 3 pounds on each side, it was found that a weight of 3 or 4 grains was sufficient to set it in rotation in either direction, and as this proportion was far beyond the practicable refinement of the measurement to be made, the friction in this wheel was regarded as negligible. By rotating the shaft with known weights in the scale pan and at the same time taking the record on the drum provided for thrust measurements, accurate indications could be obtained as to the value of each inch of record on the drum. This

process was repeated every few weeks, and the run records between successive calibrations were computed upon the basis of the mean of the two calibrations, the difference between which was in all cases very small.

In calibrating the transmission dynamometer, recourse was had to a balanced form of prony brake as shown in fig. 11. The reason for using a balanced brake rather than the usual unbalanced form was for the purpose of eliminating as far as possible all side pressure in the forward shaft-bearing and the consequent measure of an element of friction not present under operative conditions. This result was well accomplished by the means adopted, the bicycle wheel above referred to serving, as shown by the figure, to apply the pull on one side while gravity directly was employed on the other. In each case the weight added to the scale pan on the one side was the same as that on the other in order to eliminate as far as possible all side thrust. This apparatus was run with various weights in the scale pans, and registration was made upon the drum connected with the manometer for transmission dynamometer records. This afforded an accurate means of connecting records on the drum with the actual turning moment involved.

CONDUCT OF EXPERIMENTS.

In any given experiment on one propeller there are four quantities to be measured, namely: The power absorbed by the propeller, the thrust developed, the number of revolutions per minute, and the speed of advance of the propeller in undisturbed water. Explanation has already been given as to the operation of the various devices for measuring these items. A set of experiments on one propeller consisted in general of a number of runs up the scale of slip and then down again, there being usually from eight or ten to as many as fifteen or twenty runs in each series. The variations of slip were obtained by successively shifting the belt of the Evans cone pulley system along the length of the cones by regular gradations, thus covering the range of experiment desired.

The crew consisted of three men, one of whom acted as motorman while the other two cared respectively for the speed and time recorder and for the manometers. Before starting the run on each propeller, a so-called zero run was made under operating conditions of all the mechanism, except that the driving-wheels of the car were raised from the track by means of steel-jacks, the car thus remaining stationary during this "run." The shaft was also run without the propeller, but all other conditions were identical with those obtaining under ordinary operation. This zero run gave a line on each of the record drums, constituting base lines for computing the results respectively in terms of thrust and turning moment.

It was found upon trial, as indeed was anticipated, that zero records for the transmission dynamometer varied with the rotative speed at which they were made. To provide for this variation, a series of records were made showing the influence of speed, and proper corrections were applied for the

particular speeds at which the runs were made. As a rule a zero run was also made after the experiment on each propeller, thus giving a check on the first determination.

For each run there was thus obtained a thrust record, a work record, and a tape containing the three speed and time records as above outlined. These five records constitute a complete history of the operation of the propeller during the run in question, and when used with proper constants obtained by calibration, afford the necessary data for the determination of the actual forces involved, the speed of advance, and the rate of revolution of the propeller.

DEVELOPMENT OF THEORETICAL RELATIONS.

Before proceeding with the further discussion of the results, certain elementary equations must be developed, showing the nature of the relations between the principal characteristics of the propeller and the resulting performance.

For this we shall use the following notation:

$T =$ thrust.
$Q =$ torque or turning moment $=$ work $\div 2\pi N$.
$d =$ diameter of propeller.
$p =$ pitch of propeller.
$N =$ revolutions per minute.
$s =$ slip ratio.
$u =$ speed in feet per minute $= pN(1-s)$.
$e =$ efficiency.
$a =$ factor in expression for thrust which depends on slip ratio.
$b =$ " " " " " " " " pitch ratio.
$c =$ " " " " " " " " area ratio.
$x =$ " " " " torque " " " slip area.
$y =$ " " " " " " " " pitch ratio.
$z =$ " " " " " " " " area ratio.

Then quite independent of the details of propeller theory we may put

$$T = d^2(pN)^2 \times abc = d^2p^2N^2\,abc, \quad \dots\dots\dots\dots (1)$$
$$Q = d^2(pN)^3 \times xyz \div N = d^2p^3N^2\,xyz. \quad \dots\dots\dots\dots (2)$$

Equation (1) is simply equivalent to the statement that the thrust developed by the propeller will vary with the square of the diameter and with the square of the speed, and that it will depend on the slip, pitch ratio, and area ratio. The performance will, furthermore, depend in some measure on the shape of the blade and on the amount and distribution of the thickness. Since, however, the present experiments provide no means of analyzing out any such influence factors, they must be considered as constant, corresponding to the uniform elliptical shape of the blades and to the standard schedule of thickness employed. In other words, the three items in these experiments which were subjected to determinate control and which must, therefore, serve as the variables in terms of which the results may be expressed, are slip, pitch ratio,

and area ratio. The formulation of (1) expresses, therefore, in algebraic language the universally accepted relation of thrust to diameter and speed together with the general fact of dependence on these three variable items, through the factors represented by a, b, and c.

In a similar manner (2) is equivalent to the statement that the work absorbed by the propeller will vary as the square of the diameter and as the cube of the speed, and that it will be dependent also on slip, pitch ratio, and area ratio; and, furthermore, that torque varies as the work divided by the revolutions.

We then have:

$$T \times pN(1-s) = \text{useful work}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$Q \times 2\pi N \qquad = \text{total work}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

or

$$e = \frac{TpN(1-s)}{2\pi QN} = \frac{Tp(1-s)}{2\pi Q}, \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

or

$$e = \frac{(1-s)abc}{2\pi xyz} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Thus far the equations deduced are general. Next, to apply them to the present case, we shall take 100 r. p. m. as a standard or unit number of revolutions. Then, remembering that d in all cases equals one foot, we shall have in (1) and (2) for the standard revolutions:

$$T = p^2\, abc, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$Q = p^3\, xyz. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Now equations (1) and (2) show that for fixed values of d, p, a, b, c, x, y, and z (that is for a given propeller at fixed value of the slip) T and Q will vary as N^2, and hence the actual observations which were made at revolutions varying from 200 to 600 may be reduced to the standard revolutions by dividing by the square of the r. p. m. expressed in units of 100. This gives the analytical foundation for the step in the process of reduction by which the actual values of T and Q are reduced to the standard revolutions of 100 per minute, and thus made ready for plotting.

Inversely it follows that if the values of a, b, c, x, y, z, or their products abc and xyz as derived from a given observation, be assumed to hold for a larger propeller of similar form at the same slip, then (1) and (2) will serve to compute the thrust and torque, and hence the useful work and total work which the operation of such a propeller at revolutions N would involve. This gives the basis of the application of such results to the operation of design, a topic which will be considered at a later point.

As a different basis for the reduction of the observed values to standard conditions we may write (1) as follows:

$$T = d^2[p^2N^2(1-s)^2]\frac{a}{(1-s)^2}\; bc.$$

But $pN(1-s) = u$, and we have, therefore:

$$T = d^2u^2 \frac{abc}{(1-s)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Similarly we find:

$$Q = d^2pu^2 \frac{xyz}{(1-s)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

For the standard speed of 100 f. m. = 1 unit and with $d = 1$ we then have

$$T = \frac{abc}{(1-s)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$Q = \frac{pxyz}{(1-s)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

These equations show that for fixed values of d, p, a, b, c, x, y, z (that is, for a given propeller at fixed value of the slip), T and Q will vary as u^2, and hence the actual observations which were made at a nearly constant speed of 300 f. m. may be reduced to the standard speed by dividing by the square of the actual speed expressed in units of 100 f. m. This gives the analytical foundation for the step in the process of reduction by which the actual values of T and Q may be reduced to the standard speed of 100 f. m. and thus made ready for plotting.

Inversely it follows that if the values of $a \div (1-s)^2$, b, c, and $x \div (1-s)^2$ y, z, or of their products $abc \div (1-s)^2$, $xyz \div (1-s)^2$ as derived from a given observation be assumed to hold for a larger propeller of similar form at the same slip, then (9) and (10) will serve to compute the thrust and torque, and hence the useful work and total work which the operation of such a propeller at speed u would involve.

These two methods of reduction are readily connected together by noting that the relation between (7) and (11) or (8) and (12) is expressed by the factor $p^2(1-s)^2$. That is, the values given by reduction to a standard speed of 100 f. m. as in (11) and (12) when multiplied by $p^2(1-s)^2$ will give the values reduced to standard r. p. m. of 100 as indicated by (7) and (8).

In the reduction of the observations to suitable form for graphical representation either method might be followed; that is, the results might be reduced to a standard value of the r. p. m. or of the speed in feet per minute. Both methods are equally correct and either will express all the facts. In the actual case it is found that the values of T and Q, when reduced to a standard value of the r. p. m. and plotted for a given propeller on an abscissa of slip, fall nearly on straight lines, and that in groups the resulting curves are more compact and somewhat more manageable than those resulting from reduction to standard speed.

We may now turn to the consideration of certain details connected with the reduction of the observations to suitable form for general discussion and representation.

REDUCTION OF OBSERVATIONS.

This naturally comes under three heads: the tapes, the thrust records, and the moment records. The usual method of procedure was to take first the tapes for a series of runs and evaluate the records which they contained. This was done by counting and marking from the initial end of each tape the distance stations along the length of the run, these being numbered respectively from zero to eleven, corresponding with an effective run of 220 feet. Experience and observation showed that the first three of these sections were required for acceleration, which resulted in leaving eight sections, or 160 feet, for the measurement of the forces. Lines were drawn across the tape at stations numbered respectively 3 and 11, and the time and revolution records were computed for the space inclosed between these two lines. The character of the records was such that time was easily read to tenths of seconds and rotative

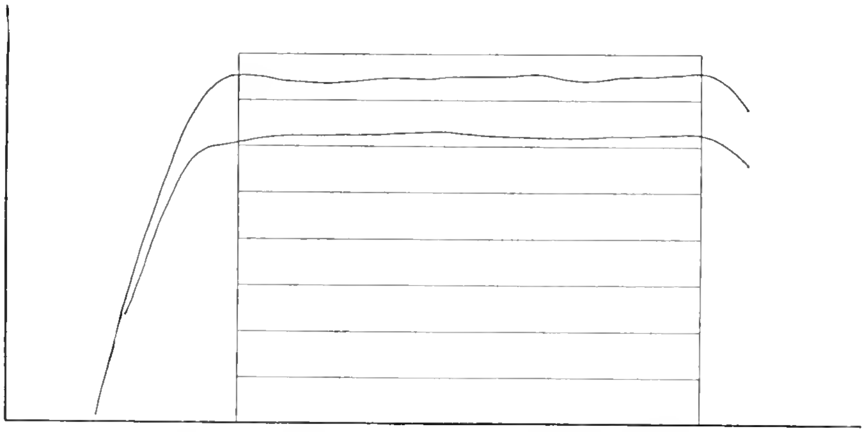


FIG. 12.—SAMPLE RECORD OF THRUST OR TURNING MOMENT.

speed to half revolutions. The total time and revolutions for the run thus determined were then entered on the data sheet and reduced to terms of speed of advance in feet per minute, and revolutions of propeller per minute.

As the next step the mean slip of the propeller for the run was determined in accordance with the relation:

$$s = \frac{pN - L}{pN},$$

where

N = revolutions for the run. L = length of run in feet.
p = pitch in feet. s = slip ratio.

Special analysis of the records also showed that speed and revolutions remained sensibly constant during the run, a fact further indicated by the generally horizontal character of the thrust and moment records between the limits included in the reduction operations. The thrust and moment records were evaluated by means of a planimeter as indicated in fig. 12.

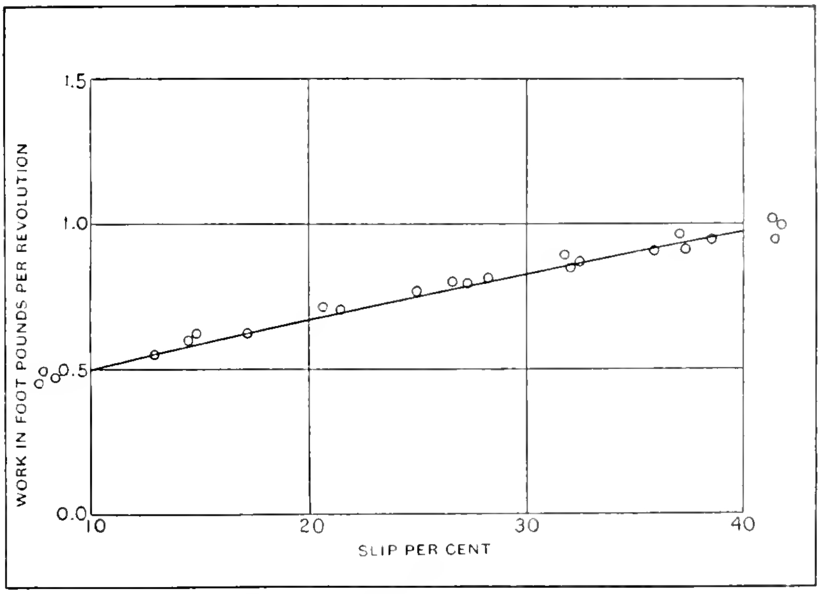


FIG. 13.—PITCH 0.9. AREA RATIO 5.

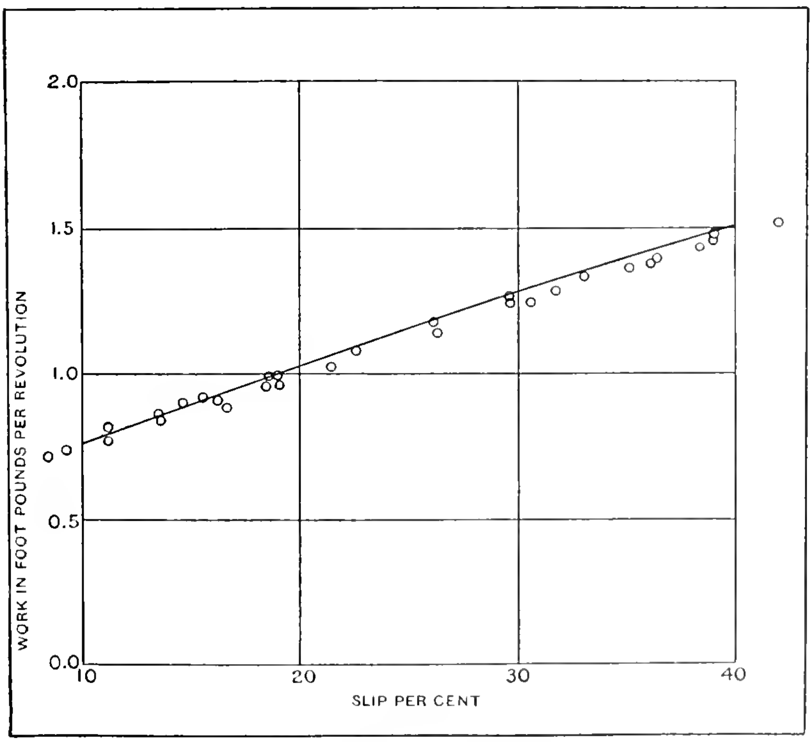


FIG. 14.—PITCH 1.1. AREA NUMBER 7.

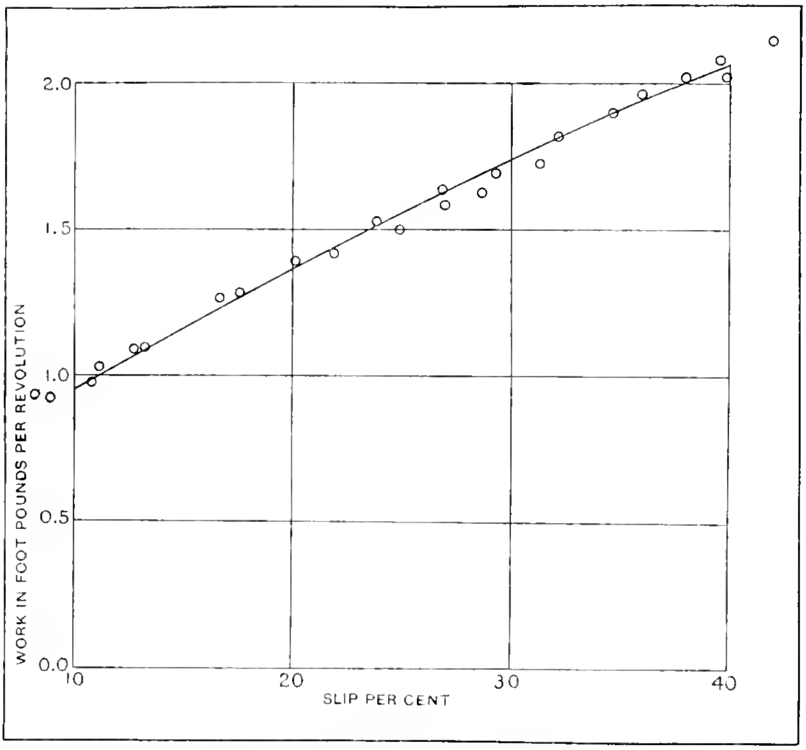


FIG. 15.—PITCH 1.3. AREA NUMBER 8.

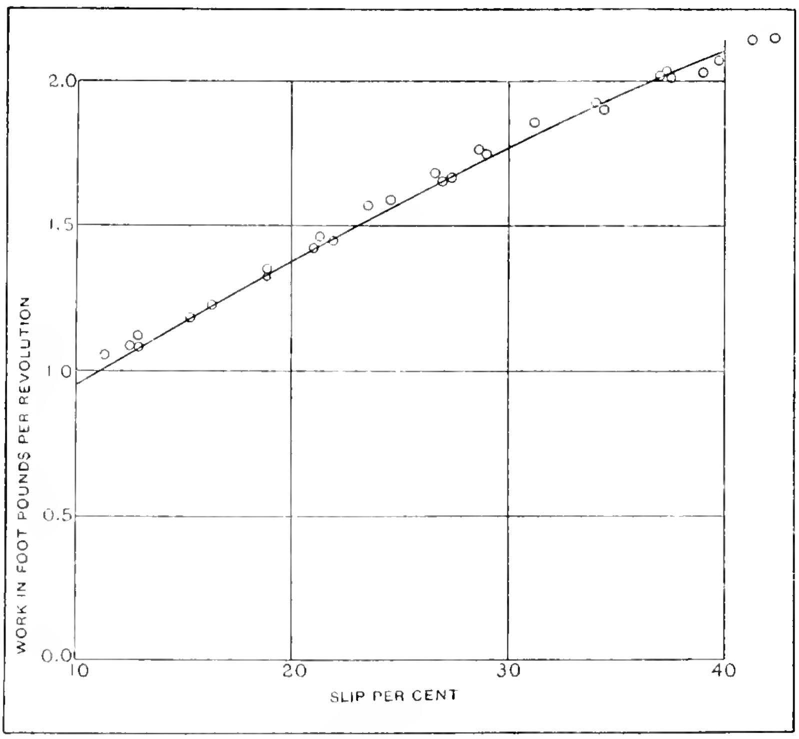


FIG. 16.—PITCH 1.5. AREA NUMBER 4.

On a sheet of records two vertical lines were drawn, perpendicular to the base line, one at the extreme end of the record, just before the drop indicating the retardation at the close, and the other near the beginning and at a distance proportioned to the entire length as the distance required for acceleration to the entire length of the run.

In order to avoid the use of the planimeter on the larger part of the area of the diagram, the sheets were ruled with lines parallel to the base and at regular distances from it with intervals of one-half inch. The instrument was then used for simply the small area contained between the actual curved or slightly wavy record, and the nearest horizontal line below. By adding the mean height of this fractional part of the area to that of its lower boundary above the base line, an accurate value of the mean ordinate for the entire area was obtained.

These ordinates were then multiplied by appropriate constants derived from calibration, thus reducing one to $T =$ thrust in pounds and the other to $W =$ work in foot-pounds per revolution $=$ torque $\times\ 2\pi$.

In order to reduce all observations to convenient standards of comparison, it was found convenient, as noted previously, to use a speed of advance of 100 feet per minute, and a rotative speed of 100 revolutions per minute. The above values being then reduced to these standard conditions, preliminary curves were drawn for each series in order to obtain graphical data from which resultant curves could be smoothed in and final results obtained.

All preliminary curves were drawn on slip ratio as an abscissa, and cross curves were then taken at four values of the slip, 10, 20, 30, and 40 per cent, such values covering the ordinary working range. Viewing the data as a whole we have then three modes of variation: according to pitch ratio, area ratio, and slip ratio. Denote these respectively by p, a, and s, thrust by T, work per revolution by W, and efficiency by e, and we have as follows:

For both T, W, and e	seven	values	of	the variable	p.
T, W, and e	"	"	"	"	a.
T, W, and e	four	"	"	"	s.

Let us now adopt a method of identifying the various curves which may result: Suppose the ordinate or quantity whose values are to be shown to be denoted by the first of a group or combination of symbols, and the corresponding abscissa by the second while following are symbols denoting the quantities remaining constant throughout the individual curve. Thus $Tsap$ will imply a curve showing T on an abscissa of s and for each of which the values of a and p will remain constant. There may then be as many of these as there are combinations of the various values of a and p, in pairs. As there are seven values of each this will give 49 pairs or 49 curves $Tsap$. It is then readily seen that we may derive in various groupings curves as indicated in the following table:

$Tsap$......49.	$Wsap$......49.	$esap$......49.
$Tpas$......28.	$Wpas$......28.	$epas$......28.
$Taps$......28.	$Waps$......28.	$eaps$......28.

These various curves were drawn for purposes of general study, and those for T and W for values reduced to both 100 feet per minute speed of advance and 100 revolutions per minute rotative speed.

With such an assemblage of curves inter-related in this manner it becomes easy to detect the departure of any one or of any value from what may be termed the normal value for the series. Minor departures from such a group value are to be expected for various reasons, chief among which are the following:

(1) Any given propeller may depart slightly in surface area or diameter from the exact values intended.
(2) Any given propeller may depart slightly from the intended pitch.
(3) Any given propeller may depart slightly from the intended thickness of blade or in the distribution of such thickness over the blade surface.
(4) Errors of observation may be made.
(5) Instrumental errors may enter into the results.
(6) Numerical errors in reduction may be made.

So far as possible these various errors have been eliminated by appropriate means. Thus, as noted, all propellers were carefully measured after the final task, and special computations made for diameter, area, and pitch. By the usual graphical methods it became then a simple matter to reduce the various results all to standard values of diameter, area, and pitch, and such corrections or reductions were made. There still remain the influence of thickness and errors of observation, of apparatus, and of reduction.

The character of the influence due to thickness is obscure and needs further examination. It is often surprisingly great, especially in shifting the location of the best efficiency relative to the slip. It is to be fairly presumed that certain minor irregularities in the actual results are traceable to the influence of this factor. Regarding errors in the apparatus it should be said that its very delicacy in certain features rendered special care needful lest disturbing elements might enter and modify the indications. Whenever any such disturbance was noted at the time the observation was repeated and presumably correct values obtained. After reaching the final reductions some few results showed such a divergence from the plainly indicated general trend as to lead to their exclusion.

Regarding errors of reduction it may be said that the actual work of measurement and numerical combination was all done with special care and was submitted to check in various ways, and it is believed that no serious errors of this character could have escaped detection.

The real purpose of the reduction of the data and of fairing up the various curves is to find the various items of the performance for a series of 49 propellers, perfect in geometrical form and with values of pitch and area ratio

exactly as indicated in table 1 and with the average or typical distribution of thickness contemplated by the sections of fig. 3. By fairing the various curves back and forth relative to variation along area, pitch, and slip, results were reached which represent continuous and regular variation in all directions and with reference to all three variables, and which thus may be fairly accepted as the results indicated for a set of propellers of perfect geometrical form and relation to each other as shown by the characteristics of table 1. The degree of departure of the final curves thus faired up and combined one with reference to all, was in most cases not great, and in only a few cases where the general demands of the body of data as a whole seemed to require a more decided departure, such points or values were practically ignored.

In order that the general character of agreement may be seen, reference may be made to figs. 13-16, where are shown representative diagrams giving original data in spots and the final reduced curves running through or near them. To reduce the number of sheets of graphical data, all final group results have been given in the form of contours or topographic surfaces as shown in figs. 61-85, Appendix I. The reduced results are given likewise in numerical form and in somewhat greater detail, including thrust in pounds, work in foot-pounds per revolution and efficiency, in table 4, Appendix II.

GENERAL OBSERVATIONS.

The general form of the curves showing thrust and work per revolution when reduced to standard revolutions and plotted on an abscissa of slip shows a nearly straight line slightly concave to the axis of the slip. This characteristic may be seen in the contours of figs. 61-67 by noting the nearly equal steps along lines for constant pitch ratio and varying slip.

In order to test the general form of these curves it seemed desirable to determine a point at 100 per cent slip for several cases, and to this end the values of the thrust and torque were determined for 10 propellers with the car stationary and hence at a slip of 100 per cent. The propellers selected were numbers 4 and 5 of pitch ratios 1.1, 1.3, 1.5, 1.7, 1.9, and the results are shown in figs. 17-20. The full line portion of the curves represents the results for thrust and work per revolution for 10, 20, 30, and 40 per cent slip taken from the main body of the data and corresponding, so far as work is concerned, with the values plotted for the same propellers in the contours of figs. 61-78. The values of thrust and work per revolution for 100 per cent slip are then plotted as shown by the small circles on the 100 per cent ordinate and the interval between 40 per cent and 100 per cent was filled in as shown by dotted line and indicating the presumable character of the curve between these known points.

The curves thus determined may presumably be accepted as showing the general character of the relation between thrust and slip on the one hand and work and slip on the other, both reduced to standard revolutions, and over the range of slip up to 100 per cent. The general character of these curves shows,

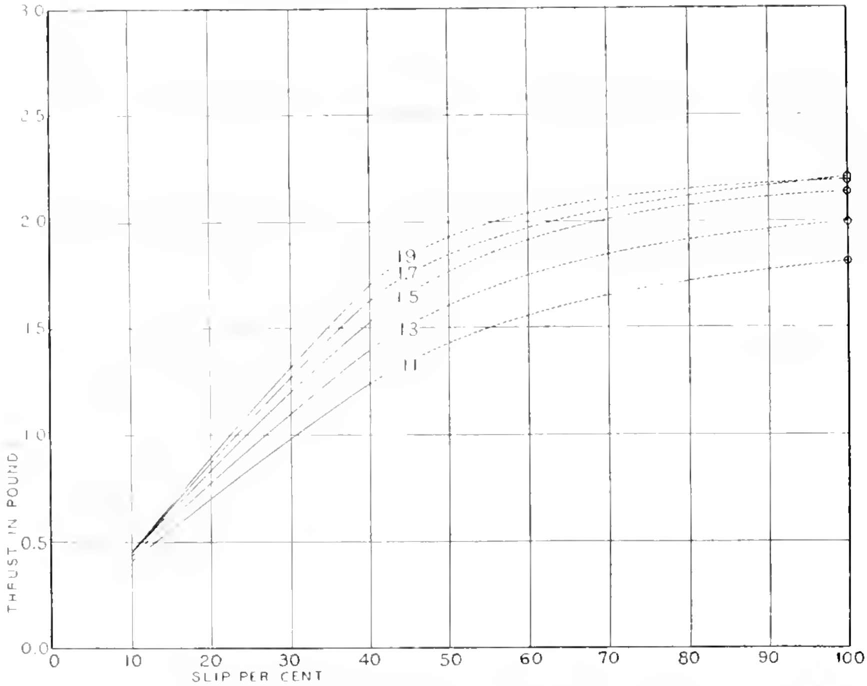


FIG. 17.—VALUES OF THRUST ON SLIP EXTENDED TO 100 PER CENT. PITCH RATIO AS MARKED ON CURVES. AREA NUMBER OF ALL PROPELLERS 4.

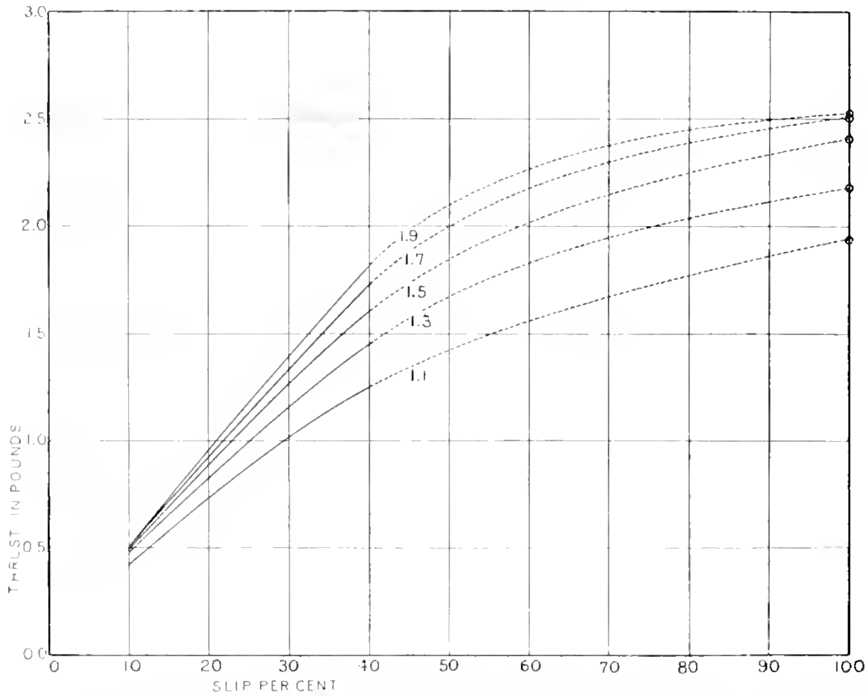


FIG. 18.—VALUES OF THRUST ON SLIP EXTENDED TO 100 PER CENT. PITCH RATIO AS MARKED ON CURVES. AREA NUMBER OF ALL PROPELLERS 5.

furthermore, very clearly that the slight departure from a straight line which appears between 10 per cent and 40 per cent must naturally be in the nature of a concavity toward the axis of slip. We may, therefore, conclude, as was shown consistently and continuously by the results throughout, that the gen-

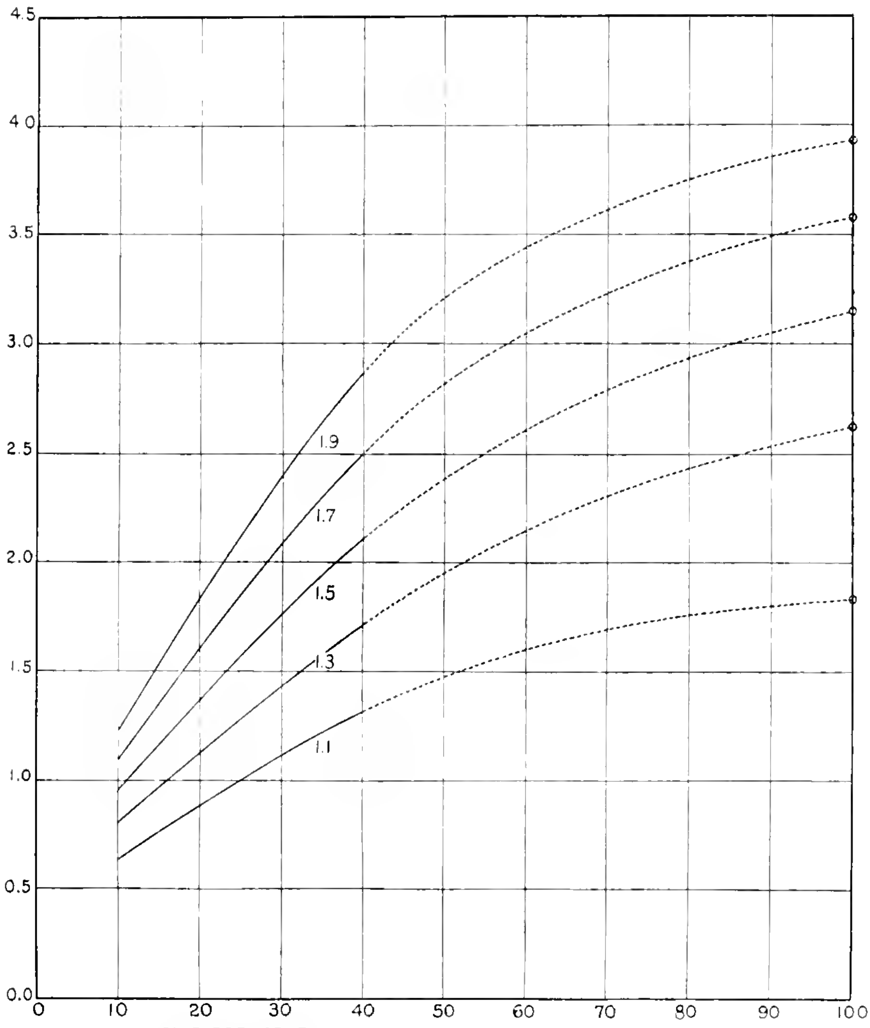


FIG. 19. VALUES OF WORK PER REVOLUTION ON SLIP EXTENDED TO 100 PER CENT. PITCH RATIO AS MARKED ON CURVES. AREA NUMBER OF ALL PROPELLERS 4.

eral form of the curves, as shown in figs. 19-20, and of the sections which would be given by cutting across the contours of figs. 61-67 in directions parallel to the slip axis, will all be nearly straight lines with a slight concavity to the axis of slip. These results are, furthermore, in accord with those furnished

by all previous experiments covering the determination of the values involved, in such form as to bring out this relation.

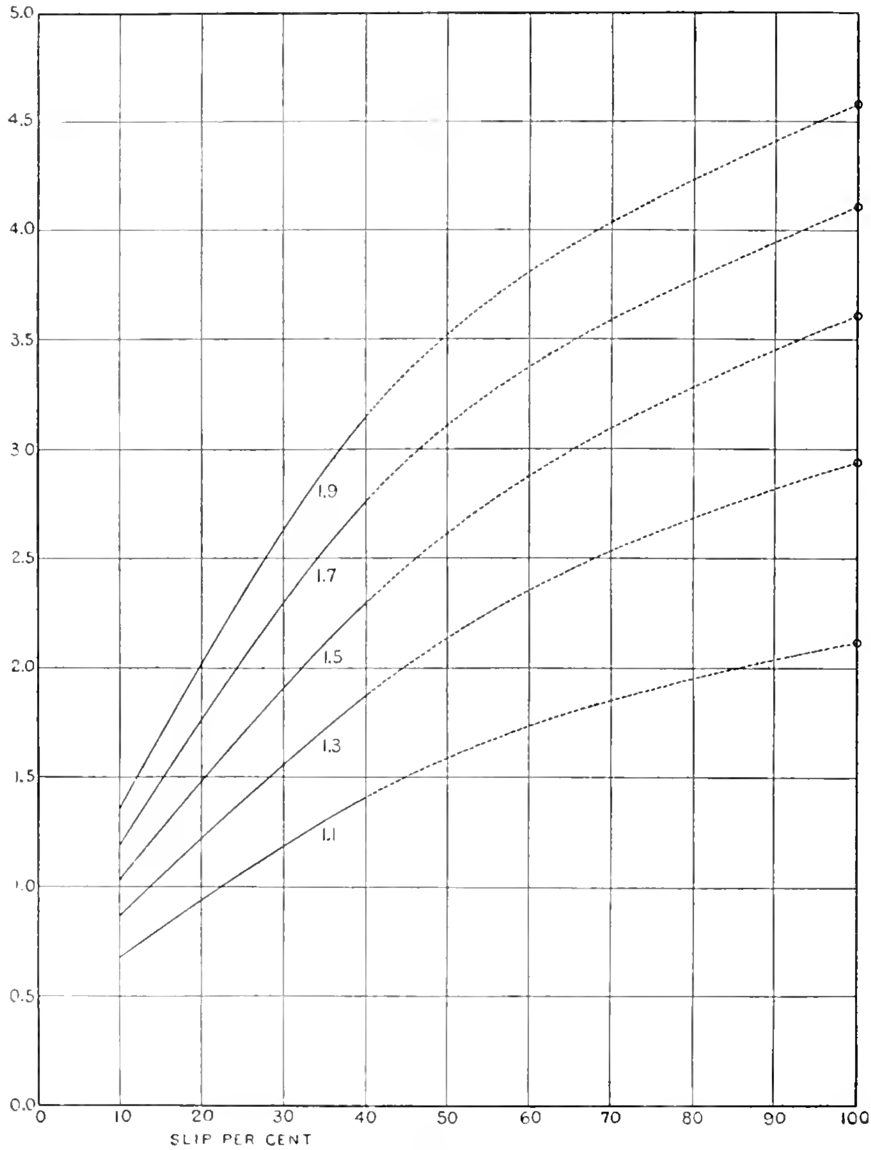


FIG. 20.

It will, furthermore, be seen from the curves of figs. 19 and 20 and from the spacing of the contours of figs. 61-67 that the slope of these lines showing work on a slip axis increases with area ratio and with pitch ratio, according to some complex law which will be a function of these two characteristics.

Turning now to the general character of the contours of figs. 61-67 along lines parallel to the axis of pitch ratio we may appreciate the nature of the law connecting work per revolution and pitch ratio at constant area and slip. It will be seen that the lines given by a section through the contours parallel to the pitch ratio axis will be up-sloping curves and not far removed from straight lines in any given case. The actual form of these lines, as would be seen readily by plotting them out in detail, shows usually a very slight convexity to the axis of pitch, though in some cases this feature was obscure and indications of a slight double curvature were found in certain cases. The general character of the results, however, points to a law nearly linear with a slope increasing with slip and with area, according to some complex law which will be a function of these two characteristics.

Turning now to the contours of figs. 68-74 we may note for constant pitch ratio and slip the relation between work per revolution and area. The curves given by sections through these contours along lines parallel to the area axis are all concave toward the latter and this indicates a decreasing rate of increase of work per revolution with area as large values of the latter are reached. This general law is in accord with theory, and with all that is known regarding the character of the forces involved in the movement of a body through a liquid. Sections through these same contours taken in the other direction, or parallel to the axis of slip, would give for constant area and pitch ratio the curves of work on varying slip as discussed above.

Likewise, the contours of figs. 75-78 when cut by sections along lines parallel to the axis of area will give curves for work on varying area for constant slip and pitch ratio, and those along lines parallel to the axis of pitch ratio will give curves for work on varying pitch ratio for constant slip and area ratio, both as already discussed above.

In general, a careful study of these contours will show more clearly than any amount of textual description the nature of the relation between work per revolution and the three characteristics of pitch ratio, area ratio, and slip ratio.

Turning now to the contours for efficiency shown in figs. 79-85 we find indications of the highest importance in connection with the general problem of propeller design. The principal conclusions may be noted as follows:

(*a*) The general character of the efficiency curve for varying slip, the pitch ratio and area being constant, is well known and would be given by sections cut from the contours along lines parallel to the slip axis. Such curves as shown by the contours indicate a low value of the efficiency for low values of the slip rising to a maximum in the neighborhood of 20 or 25 per cent and then falling off more gradually for increasing values of the slip.

(*b*) In general there is indication that the propeller of very small area reaches its efficiency at a relatively high slip, usually 25 per cent or over, while that of large area reaches its best value at about 20 per cent and slightly below.

(c) The highest values seem, furthermore, to be only slightly dependent on either area or pitch ratio, by far the larger number of the propellers under test reaching at some value of the slip an efficiency close to 70 per cent, with, in general, the best results for fairly high values of the pitch ratio and moderate areas.

(d) In general, propellers of high pitch ratios reach their best efficiency at a somewhat higher slip than those of low pitch ratio.

(e) For low values of the slip such as 10 to 15 per cent and low pitch ratios, increase of area is accompanied first by increase of efficiency and then by some falling off, while for higher values of the pitch ratio and low slip there is a general improvement of the efficiency up to some value at which it remains nearly stationary.

(f) For high values of the slip there is, in general, a continuous, though slow, decrease in efficiency with area through the range under investigation.

(g) Intermediate between these extremes we find for moderate values of the slip but slight change of efficiency with area.

(h) For propellers lying within the more common ranges of pitch ratio and area and operating at values of the slip between 20 and 30 per cent, it appears that the efficiency will vary but slightly over the mid-field range thus indicated, and that any proportions corresponding to operation within these general limits may be freely chosen without fear of any important drop in efficiency below the limits of 68 or 70 per cent.

PROPELLER DESIGN.

We shall now turn briefly to the application of such experimental results to the problem of propeller design. Let

Q = turning moment. F = work in foot-pounds.

P = work in terms of horse-power unit.

Then reproducing (2) of the section on theory,

$$Q = d^2p^3N^2(xyz), \qquad (1)$$

$$F = 2\pi NQ = d^2(pN)^3(2\pi xyz). \qquad (2)$$

$$P = d^2(pN)^3(2\pi xyz) \div 33{,}000. \qquad (3)$$

Now in order to apply the results obtained from the study of experimental propellers to the problem of design we must assume in effect that (2) or (3) is general in its application to all sizes of propeller and that the values of x, y, z, are independent of size. The rationale of the process consists then in the determination of the value of (xyz) or of $2\pi xyz$ from the model propeller and for the given conditions of slip, pitch ratio, and area ratio, and then in the use of the same value in (3) with appropriate values of the other factors.

Now in (2) we have the value of the work in foot-pounds for any propeller. Dividing by N we have for work per revolution,

$$F \div N = d^2p^3N^2(2\pi xyz). \qquad (4)$$

To apply this first to the model propellers we remember that $d = 1$ and N as reduced for plotting = 100. Then if we let f denote the pitch ratio we

shall have $f = p \div 1 = p$. Substituting these values we have from (4)

$$W = F \div N = f^3 10^4 (2\pi xyz). \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Now as plotted in the diagrams the ordinate in all cases is $W = F \div N$ or work in foot-pounds per revolution. We have, therefore,

$$2\pi xyz = \frac{W}{f^3 10^4}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Then substituting this in the general equation (3) we have

$$P = \frac{d^2 (pN)^3 W}{f^3 \times 33 \times 10^7}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

For numerical convenience we may now change units as follows: For d and p use a unit of 10 feet. For N use a unit of 10 r. p. m. Then if we denote the new values of diameter pitch and revolutions by d_1, p_1, and N_1, we shall have $d = 10d_1$, $p = 10p_1$, and $N = 10N_1$. Substituting these values in (7) we have:

$$P = \frac{10 d_1^2 (p_1 N_1)^3 W}{33 f^3} = .303 d_1^2 \left(\frac{p_1 N_1}{f}\right)^3 W. \quad \ldots\ldots\ldots\ldots (8)$$

It will be noted that P denotes the power actually delivered to and absorbed by the propeller. This will be less than the I. H. P. by the amount absorbed in the mechanical friction of the engine and shafting, including the thrust block and stern tube bearings. This in ordinary cases will range from 10 to 15 per cent of the I. H. P. and the value of P in such cases will, therefore, range from 85 to 90 per cent of the I. H. P.

It should be also noted, as shown in table 1, that with sufficient accuracy for all engineering purposes we have:

Area ratio = area number $\times$ .09; or Area number = area ratio $\div$ .09.

Reference should also be made to the double aspect under which the term slip may appear in cases of actual design—slip with reference to the general outlying body of water, called the *apparent* slip, and that with reference to the following wake at the stern and in which the propeller must work, and called the *true* slip. Denoting these two values respectively by s_1 and s_2 they are usually connected by the equation:

$$(1 - s_1) = (1 + w)(1 - s_2).* \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Where w is the average or effective wake speed expressed as a percentage of the speed of the stern of the ship through the wake. In any given case if w be known or assumed,† the relation between the true and apparent slip is immediately found.

Also if u = speed in knots and remembering that s_1 has relation to external speed we have:

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* See Author's Resistance and Propulsion of Ships, page 227.

† *Ibid.*, pages 263, 285.

$$u = \frac{pN(1 - s_1)}{101.3}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

and

$$PN = \frac{101.3u}{(1 - s_1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

With these various facts in mind the application of the experimental results to design problems may be readily shown by a few examples.

(a) Let us assume

I. H. P. = 5200. Speed = u = 17 knots. Friction power = 13 per cent.

Hence

$$P = .87 \times 5200 = 4524.$$

Next assume or take on trial,

Pitch ratio = 1.3. True slip ratio = .26.
Area ratio = .40. Area number = 4.44.

Also assume wake factor

$$w = .14.$$

Then from (9)

$$(1 - s_1) = .844. \qquad pN = \frac{17 \times 101.3}{.844} = 2040. \qquad p_1N_1 = 20.4.$$

Then from fig. 70, for slip = .26 and area No. 4.4, we readily estimate W = 1.38; and substituting this we have

$$4524 = .303 d_1^2 \left(\frac{20.4}{1.3}\right)^3 \times 1.38,$$

or

$$d_1^2 = 2.80. \qquad d^2 = 280. \qquad d = 16.73 \text{ feet.}$$

Then

$$p = 1.3 \times 16.73 = 21.75 \text{ feet.} \qquad N = 2040 \div 21.75 = 93.8.$$

(b) Suppose twin screws, 8000 I. H. P. speed = 30 knots. Let friction as before = 13 per cent. Then for one propeller

$$P = .87 \times 4000 = 3480.$$

Assume

$$w = .06. \qquad s_2 = .24.$$

Then

$$(1 - s_1) = .806. \qquad p_1N_1 = 37.70.$$

Assume for trial,

f = 1.6. area ratio = .45. area number = 5.

Then

$$W = 1.82. \qquad d_1^2 = \frac{4.196 \times 3480}{.303 \times 53.583 \times 1.82} = .4940.$$

$$d = 7.03 \text{ feet.} \qquad p = 11.25 \text{ feet.} \qquad N = 3770 \div 11.25 = 335.$$

If f be taken at 1.5 instead of 1.6 the design will be modified as follows: We shall have

$$W = 1.67. \qquad d_1^2 = .4333.$$

$$d = 6.58 \text{ feet.} \qquad p = 9.87 \text{ feet.} \qquad N = 382.$$

Suppose again we have the same data as in (a) but fix the revolutions at 120 we may then proceed as follows:

With the same value for pN we have $p = 2040 \div 120 = 17$ feet.

Assume for trial a diameter of 15 feet: This gives $f = 17 \div 15 = 1.13$. We then have:

$$4524 = .303 \times (1.5)^2 \times \left(\frac{20.4}{1.13}\right)^3 \times w.$$

Whence

$$w = 1.078.$$

An examination of the topographic surface curves will show that this corresponds very nearly to area No. 4 or to area ratio .36, thus indicating a propeller as follows:

$d = 15$ feet. $p = 17$ feet. $N = 120$. Area ratio $= .36$.

In case the value of W is such that it can not be found on the diagrams, or if found, would imply such a low efficiency as indicated by figs. 79-85 as to be unsatisfactory, then the assumption regarding diameter must be changed and thus by trial and error in the usual manner, a final satisfactory combination may be developed.

In a similar manner we may deal with the case of diameter fixed in advance instead of revolutions.

EXPERIMENTS WITH PROPELLERS IN TANDEM.

In the early days of the application of the steam turbine to marine propulsion, use was commonly made of two or even three propellers on one shaft. This was for the purpose of obtaining a suitable amount of blade area in spite of the small diameter which naturally goes with the high rotative speed of the turbine. With but slight data to guide in the selection of the characteristics of the two propellers intended to be thus related, they were not uncommonly chosen the same in all features. Since, however, the two propellers will be working in water quite differently related to the boat, the after one working in the wake of the forward, it is clear that the two propellers should not have the same characteristics, and we might naturally expect that the best results would be yielded with a combination in which the pitch of the after propeller is greater than that of the forward.

With a view to the enlightening of these points a number of tests were made with tandem propellers in various combinations. These are shown in the accompanying table which will serve to indicate the nature of the various combinations investigated.

Thirty-seven combinations were examined, numbered for convenience as in the table. In Nos. 1 to 26 inclusive the propellers were mounted on the shaft at a distance apart of 12 inches or of one diameter, and the blades of the two propellers were lined up in the same fore-and-aft planes. In Nos. 27 to 32

TABLE 3.—COMBINATIONS OF TANDEM PROPELLERS.*

Combination.	Forward.	Aft.	Virtual pitch.	Combination.	Forward.	Aft.	Virtual pitch.
No. 1...	.9—4	1.5—5	1.25	No. 19...	1.5—5	.9—4	1.25
2...	.9—5	1.5—5	1.23	20...	1.5—5	1.3—4	1.42
3...	1.1—4	1.5—5	1.34	21...	1.7—4	1.5—5	1.59
4...	1.1—4	1.7—5	1.44	22...	1.7—5	1.5—5	1.61
5...	1.1—4	1.9—5	1.55	23...	1.9—4	1.5—5	1.67
6...	1.1—4	2.1—4	1.59	24...	1.9—5	1.5—5	1.71
7...	1.1—4	2.1—5	1.64	25...	2.1—4	1.5—5	1.76
8...	1.1—5	1.5—5	1.32	26...	2.1—5	1.5—5	1.79
9...	1.3—2	1.5—5	1.44	27...	1.5—5	1.7—4	1.59
10...	1.3—3	1.5—5	1.43	28...	1.7—4	1.5—5	1.59
11..	1.3—4	1.5—5	1.42	29...	1.5—4	1.5—5	1.51
12...	1.3—5	1.5—5	1.41	30...	1.5—5	1.5—4	1.51
13...	1.3—6	1.5—5	1.40	31...	1.3—4	1.5—5	1.42
14...	1.3—7	1.5—5	1.39	32...	1.5—5	1.7—4	1.42
15...	1.3—8	1.5—5	1.38	33...	.9—4	1.5—5	1.25
16...	1.5—4	1.5—5	1.51	34...	1.5—5	.9—4	1.25
17...	1.5—5	1.7—4	1.59	35...	1.5—4	1.5—5	1.51
18...	1.5—5	2.1—4	1.76	36...	1.5—5	2.1—4	1.76
				37..	2.1—4	1.5—5	1.76

inclusive the distance was maintained the same but the blades were so lined up as to stand in fore-and-aft planes at 45 degrees with each other. In Nos. 33 to 37 inclusive the blades were as in Nos. 1 to 26 but the distance was reduced to 8 inches. In the reduction of the results of these combinations it became necessary to adopt an arbitrary definition of pitch and slip. For this purpose the following definition was chosen:

Let p_1, A_1 be the pitch and area of one propeller and p_2, A_2 those of the other. Then the pitch of the combination was taken as:

$$p = \frac{p_1A_1 + p_2A_2}{A_1 + A_2}.$$

It should be noted that in these combinations p_1 and p_2 are taken as the actual values shown in table 2 and not the nominal values to which the general results with single propellers were reduced.

These resultant values of the pitch are shown in the table and served to determine the values of the slip ratio upon which the work and efficiency are plotted in figs. 21-57. It will be of course realized that actual numerical

* Propellers are indicated by nominal pitch and area number.

values of the work and of the efficiency are independent of the particular definition or standard relative to which pitch and slip are determined, and that a change in this standard will simply have the result of changing the location of such values along the axis of slip. With this general understanding reference may be made to figs. 21-57 where are given the results for work in foot-pounds per revolution and for efficiency, both plotted on slip as above defined. Those results may with advantage be compared together in various groupings, the character of which may readily be selected from the presentation of table 3. Some of the more interesting of such groupings may, furthermore, be noted as follows:

(*a*) The pairs (1), (16)—(11), (17)—(19), (21)—(20), (25)—(27), (28)—(31), (32)—(33), (34)—(36), (37) show the relative effect of low pitch forward versus high pitch forward, indicating clearly the superiority of the former, as may indeed be naturally expected for reasons as noted above.

(*b*) The group (3), (4), (5), (7) shows the effect of continually increasing the pitch of the following propeller.

(*c*) The group (9), (10), (11), (12), (13), (14), (15) shows the effect of continually increasing the area of the leading propeller.

(*d*) The groups (21), (23), (25) and (22), (24), (26) show the effect of increasing the pitch of the leading propeller when it is in all cases larger than that of the following propeller.

(*e*) Such pairs as (1), (2)—(6), (7)—(21), (22)—(23), (24)—(25), (26) indicate the nature of the effects due to slight changes in the area of one of the components.

(*f*) The pairs (11), (27)—(18), (29)—(19), (31) show the effects due to change in the fore-and-aft planes in which the blades stand and suggest a slightly improved value of the efficiency for the 45-degree arrangement.

(*g*) The pairs (1), (33)—(16), (34)—(18), (35)—(20), (36)—(25), (37) show the results of reducing the distance between the propellers from 12 inches to 8 inches or from one diameter to two-thirds diameter, the results in general indicating substantially the same values for the smaller as for the larger distance.

The efficiency is seen in all cases to be poor—distinctly lower than for the individual propellers of which combinations are formed. The best efficiencies on the whole were realized with combinations 27 to 32 with the blades of the two propellers in planes at 45 degrees with each other, though the advantage is but slight compared with the best of the other pairs, such as 13, 18, 19, 35. The poorest results were met with in combinations with the higher pitch leading, as in 16, 23, 25, and in these cases the values of the efficiency increased as a rule with slip. So far as these results may be used as a basis of inference we may hardly expect for the usual combinations of propellers a value of the efficiency much exceeding 60 per cent, or about 10 per cent less than the best results with the two components individually.

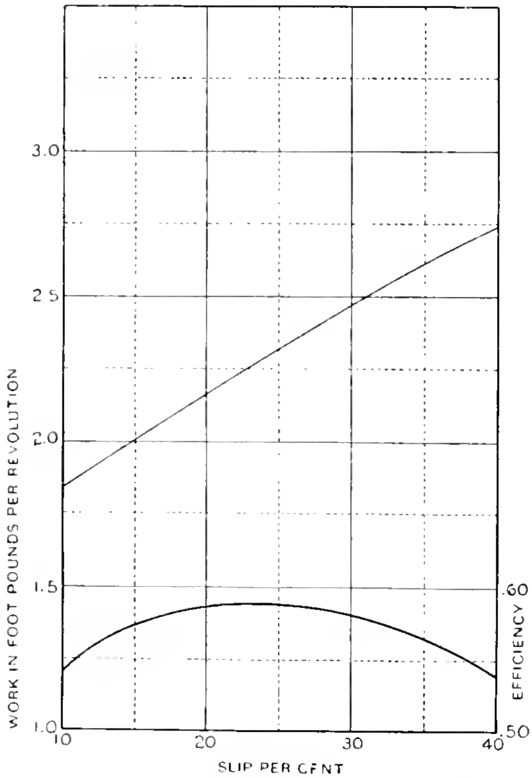

FIG. 21.—TANDEM PROPELLERS. COMBINATION NO. 1.

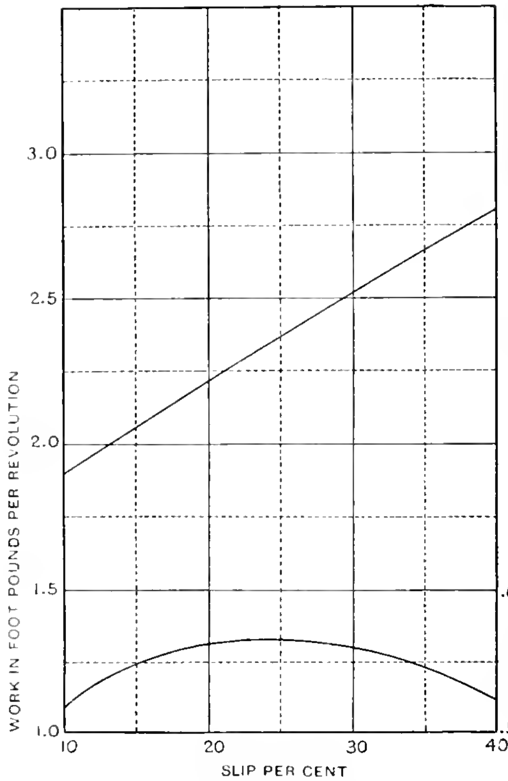

FIG. 22.—TANDEM PROPELLERS. COMBINATION NO. 2.

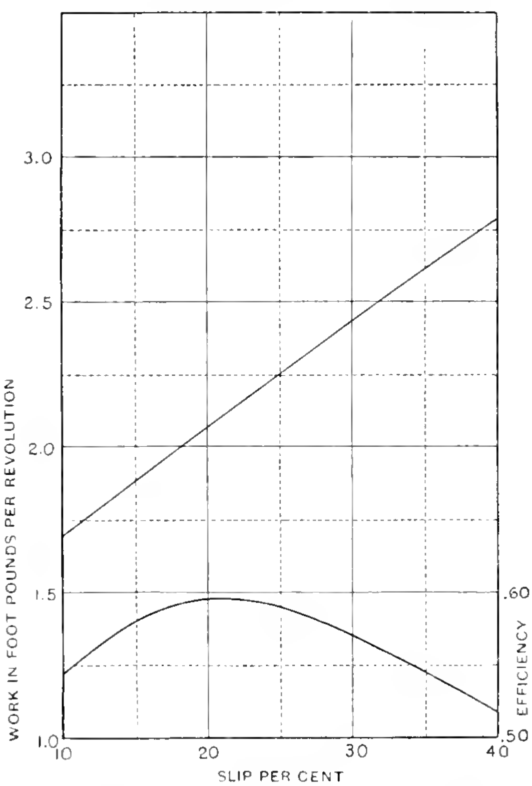

FIG. 23.—TANDEM PROPELLERS. COMBINATION NO. 3.

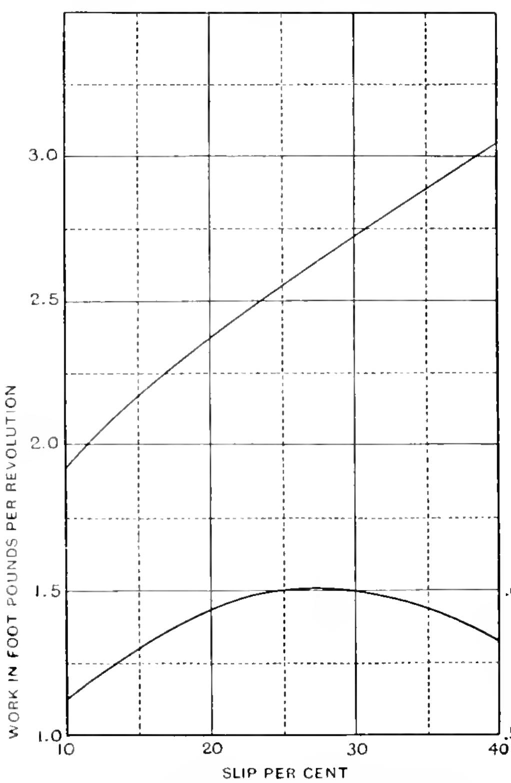

FIG. 24.—TANDEM PROPELLERS. COMBINATION NO. 4.

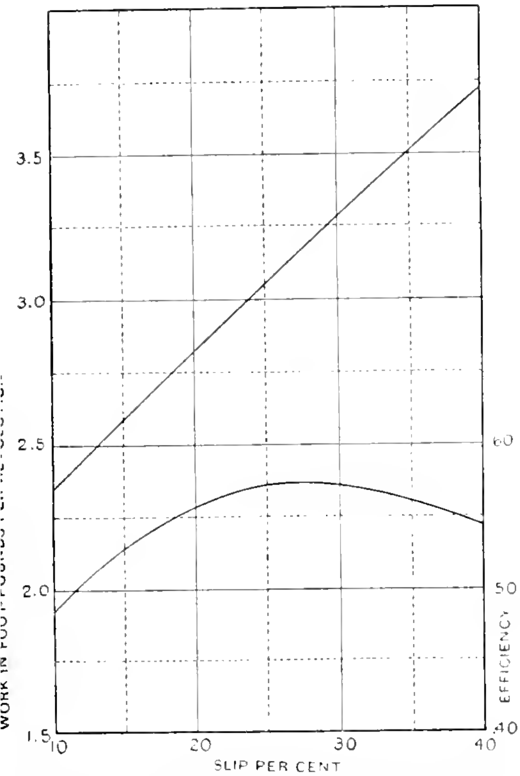

Fig. 25.—Tandem Propellers. Combination No. 5.

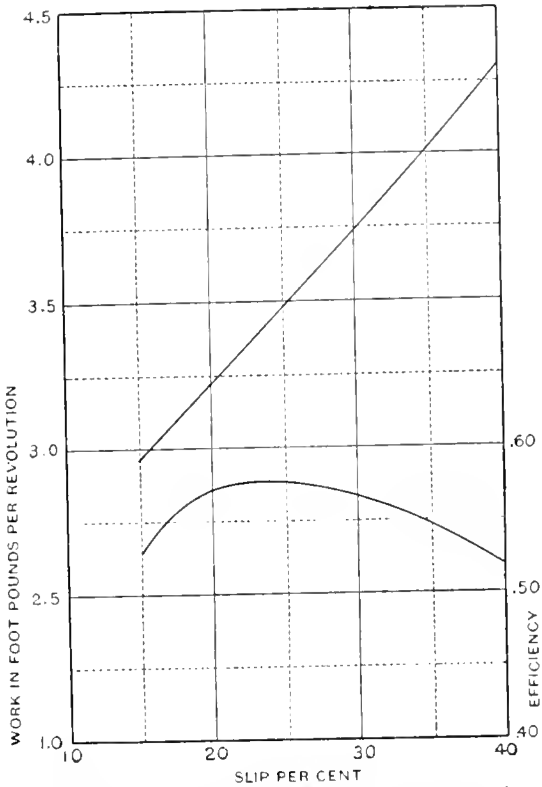

Fig. 26.—Tandem Propellers. Combination No. 6

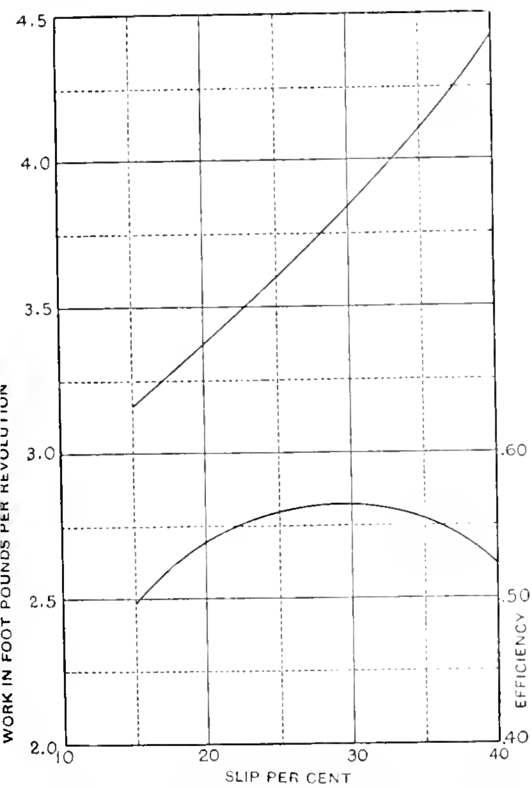

Fig. 27.—Tandem Propellers. Combination No. 7.

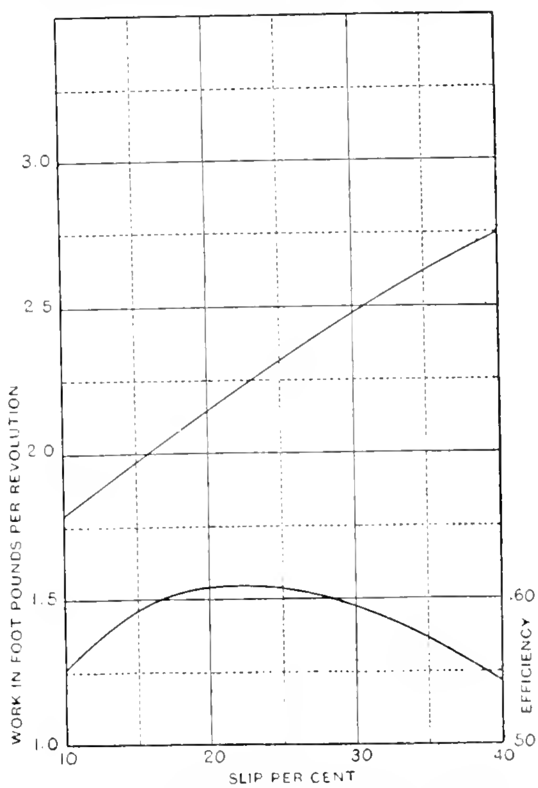

Fig. 28.—Tandem Propellers. Combination No. 8.

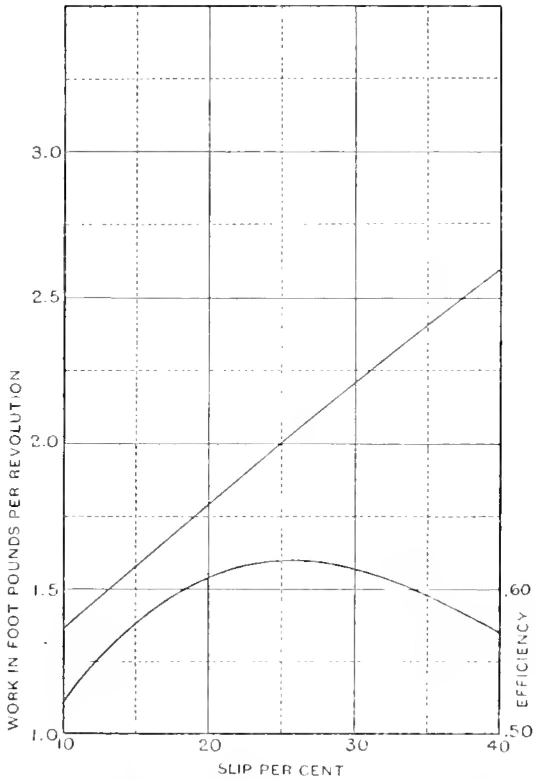

Fig. 29.—Tandem Propellers. Combination No. 9.

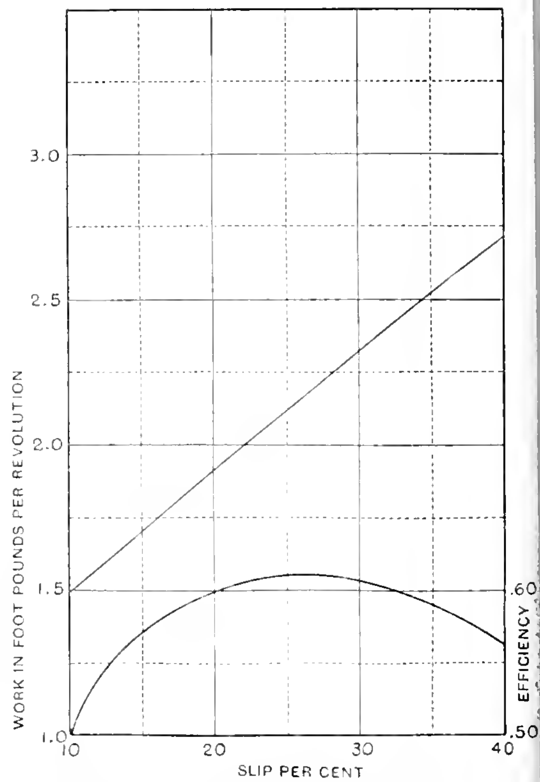

Fig. 30.—Tandem Propellers. Combination No. 10.

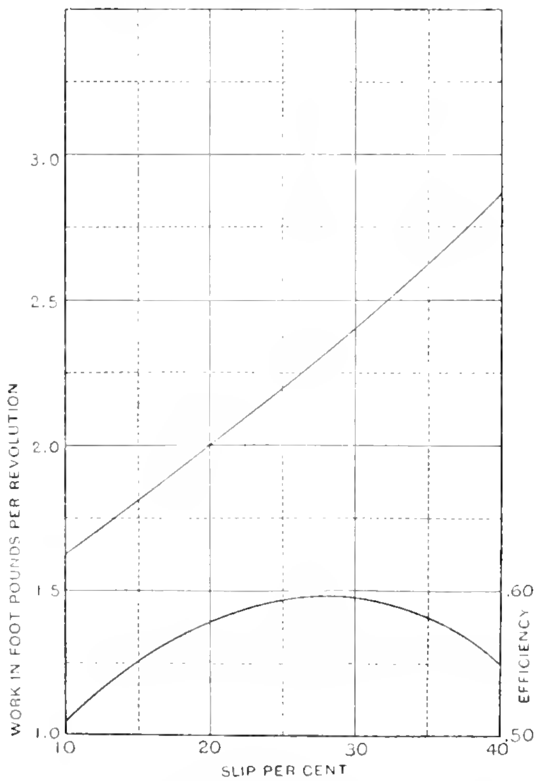

Fig. 31.—Tandem Propellers. Combination No. 11.

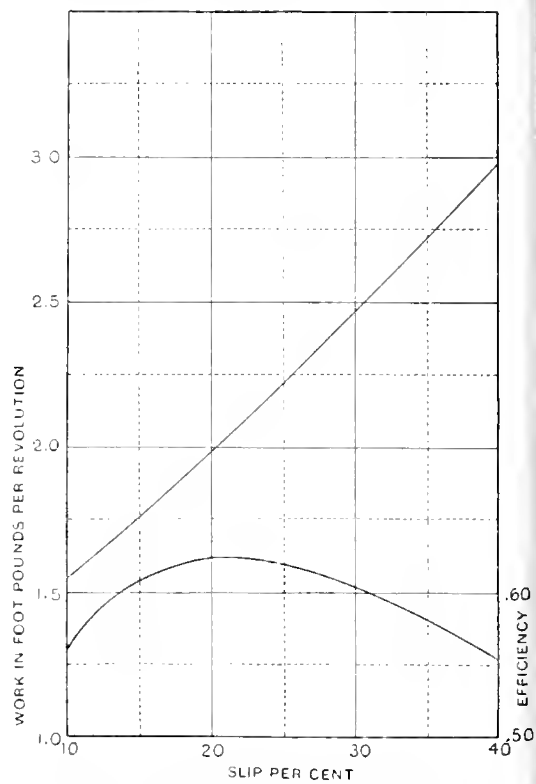

Fig. 32.—Tandem Propellers. Combination No. 12.

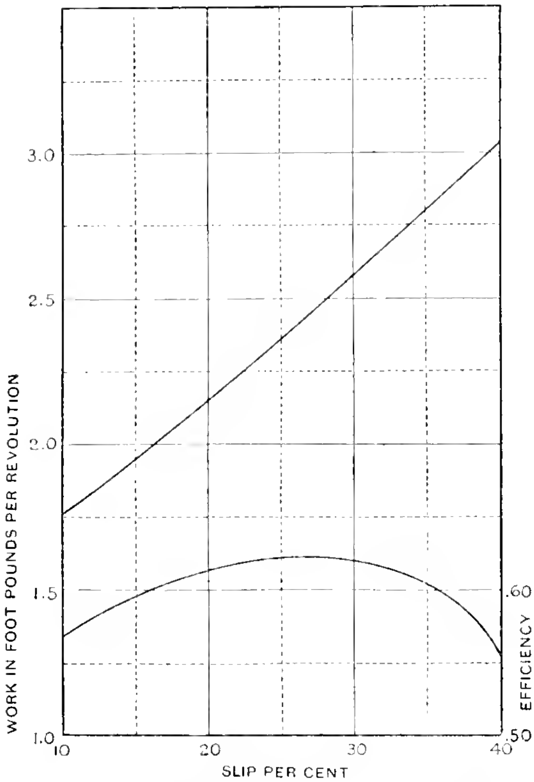

Fig. 33.—Tandem Propellers. Combination No. 13.

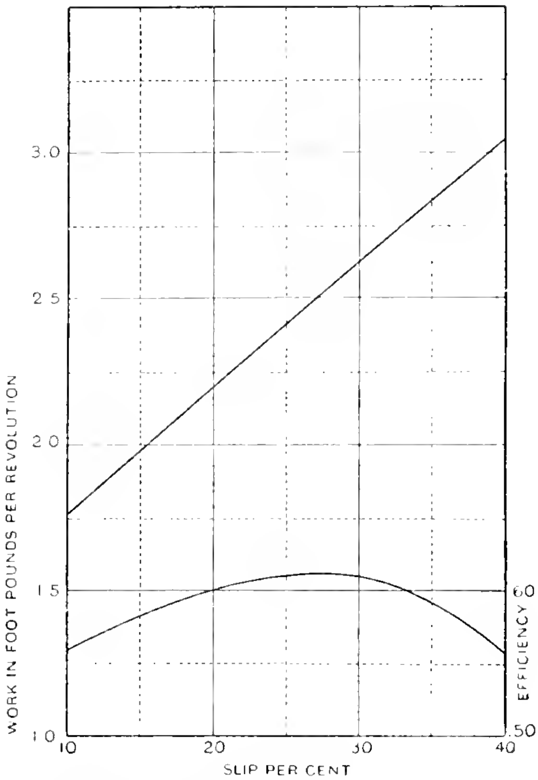

Fig. 34.—Tandem Propellers. Combination No. 14.

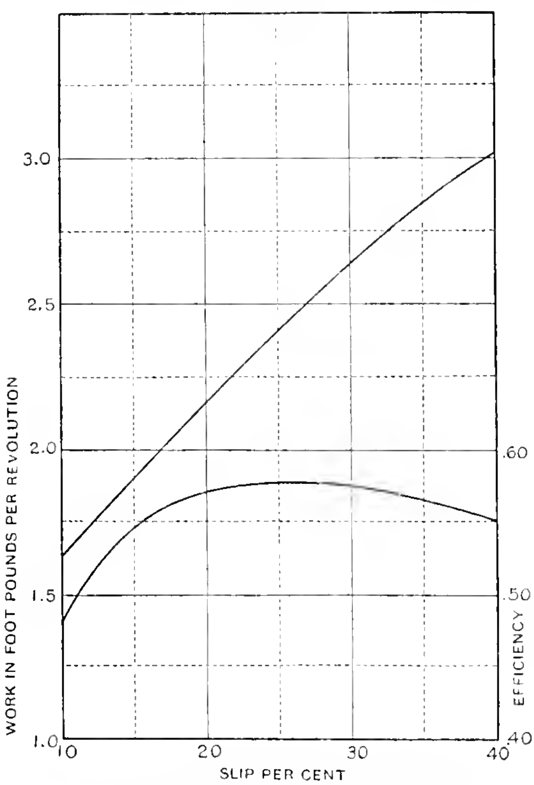

Fig. 35.—Tandem Propellers. Combination No. 15.

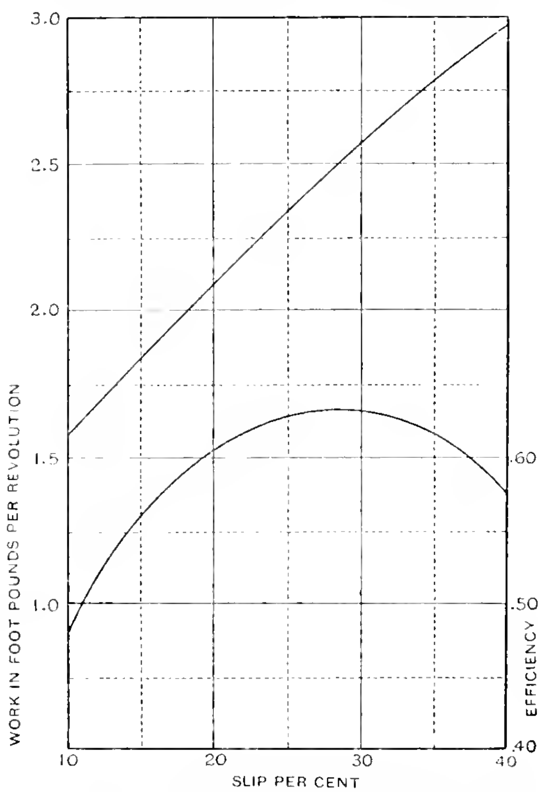

Fig. 36.—Tandem Propellers. Combination No. 16.

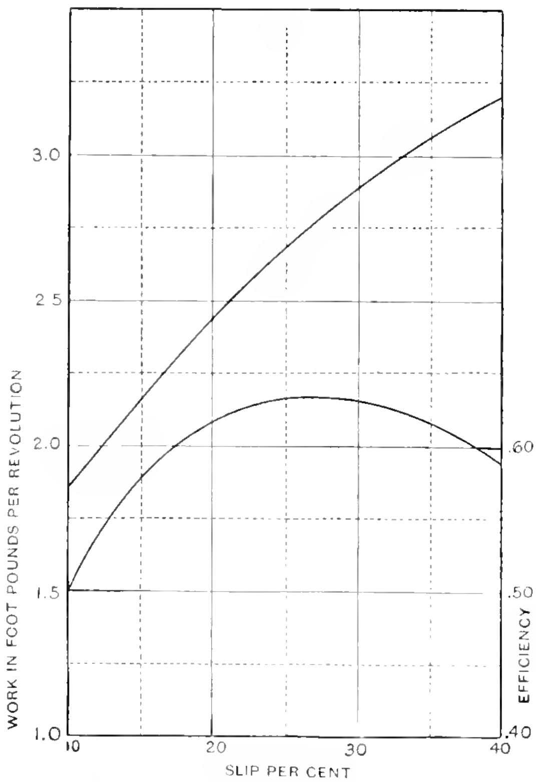

FIG. 37.—TANDEM PROPELLERS. COMBINATION No. 17.

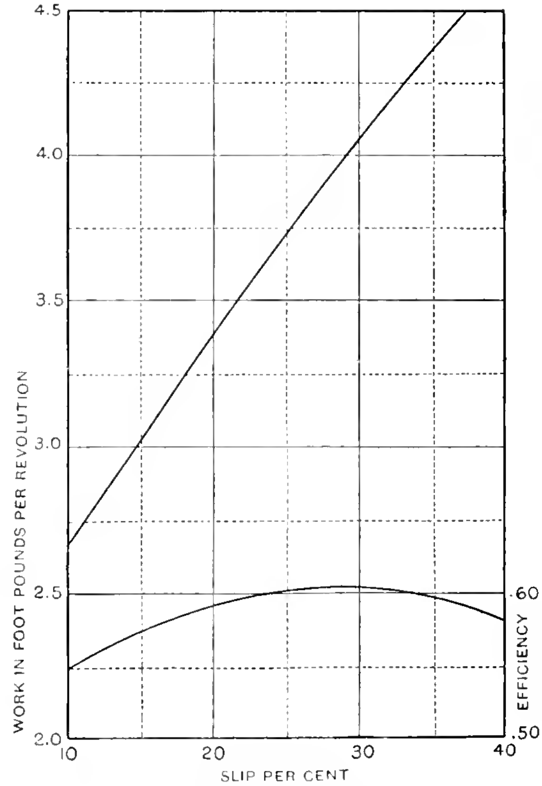

FIG. 38.—TANDEM PROPELLERS. COMBINATION No. 18.

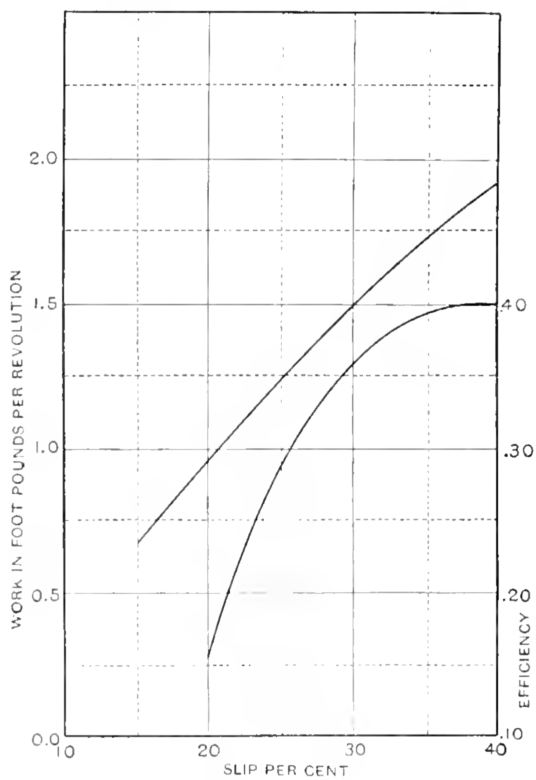

FIG. 39.—TANDEM PROPELLERS. COMBINATION No. 19.

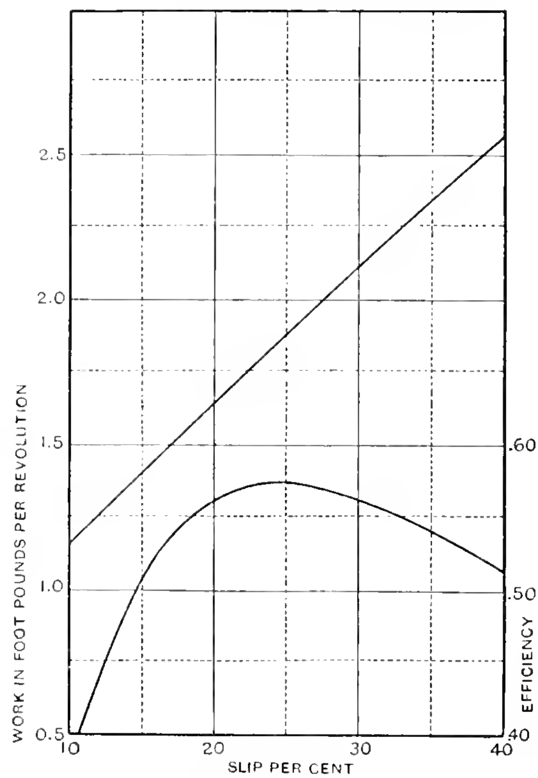

FIG. 40.—TANDEM PROPELLERS. COMBINATION No. 20.

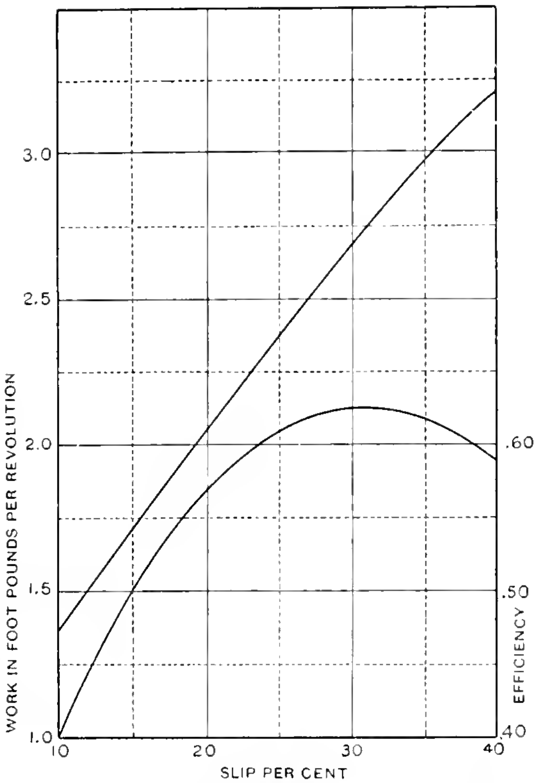

FIG. 41.—TANDEM PROPELLERS. COMBINATION NO. 21.

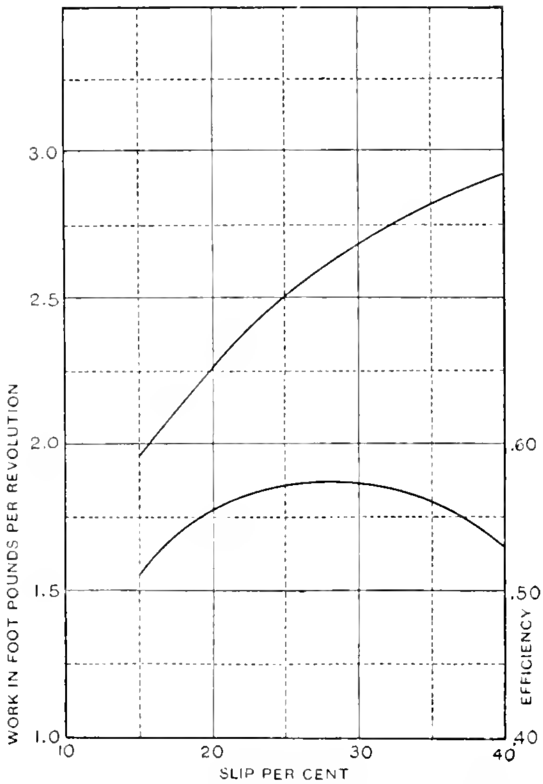

FIG. 42.—TANDEM PROPELLERS. COMBINATION NO. 22.

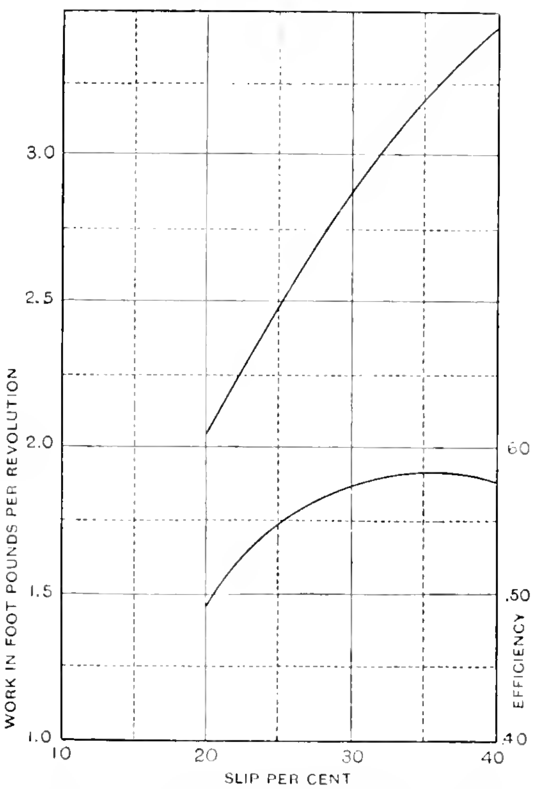

FIG. 43.—TANDEM PROPELLERS. COMBINATION NO. 23.

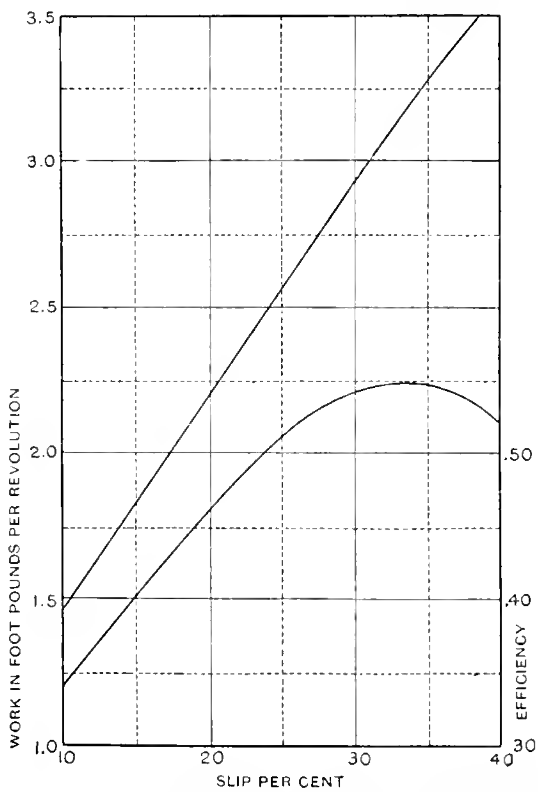

FIG. 44.—TANDEM PROPELLERS. COMBINATION NO. 24

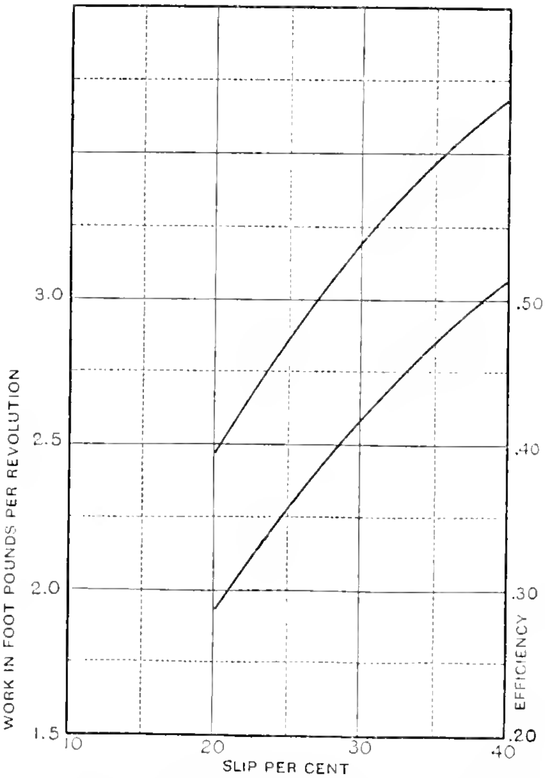

FIG. 45.—TANDEM PROPELLERS. COMBINATION No. 25.

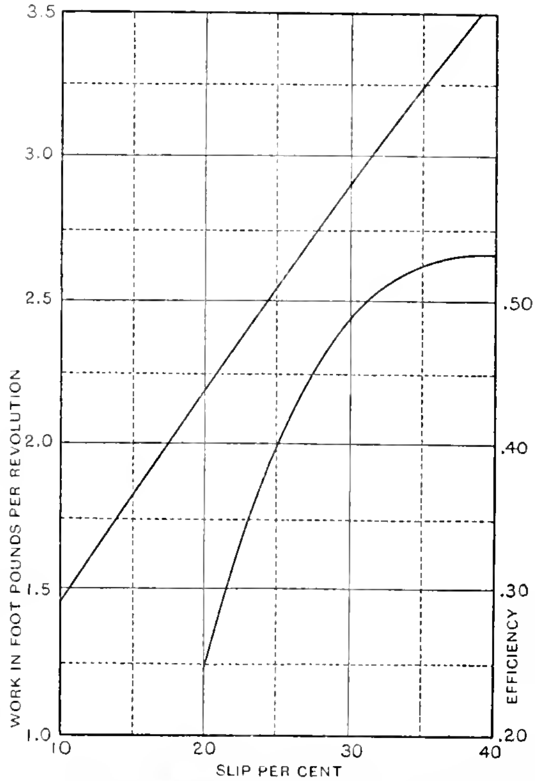

FIG. 46.—TANDEM PROPELLERS. COMBINATION No. 26.

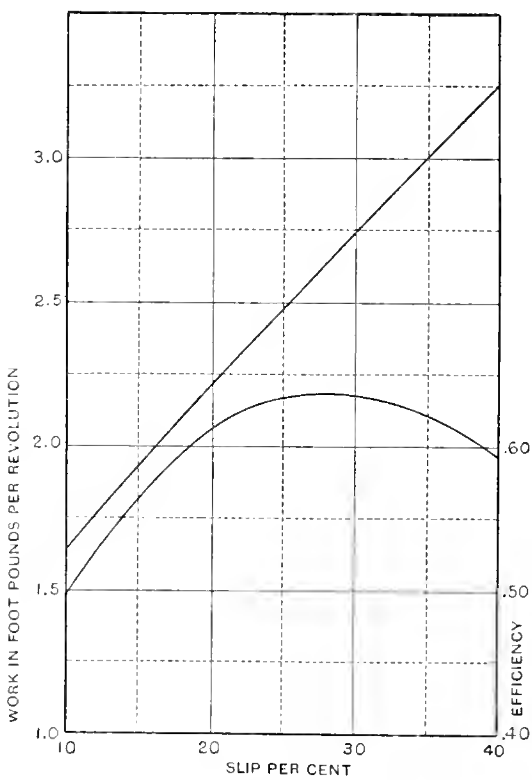

FIG. 47.—TANDEM PROPELLERS. COMBINATION No. 27.

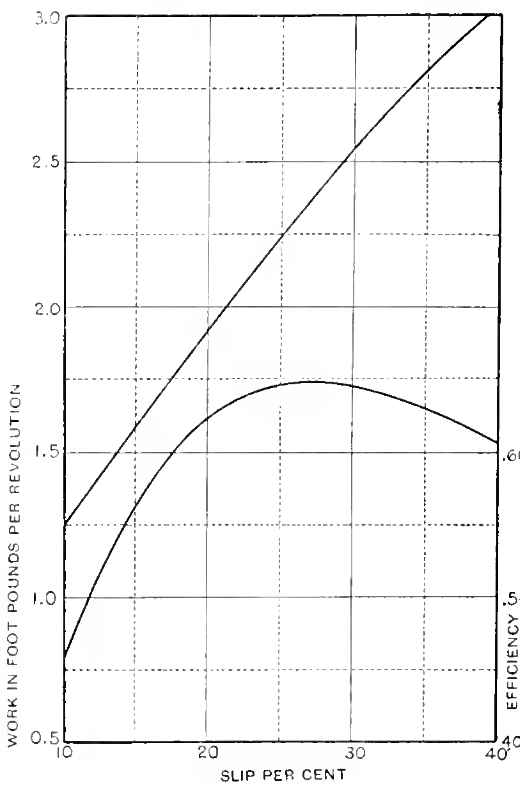

FIG. 48.—TANDEM PROPELLERS. COMBINATION No. 28.

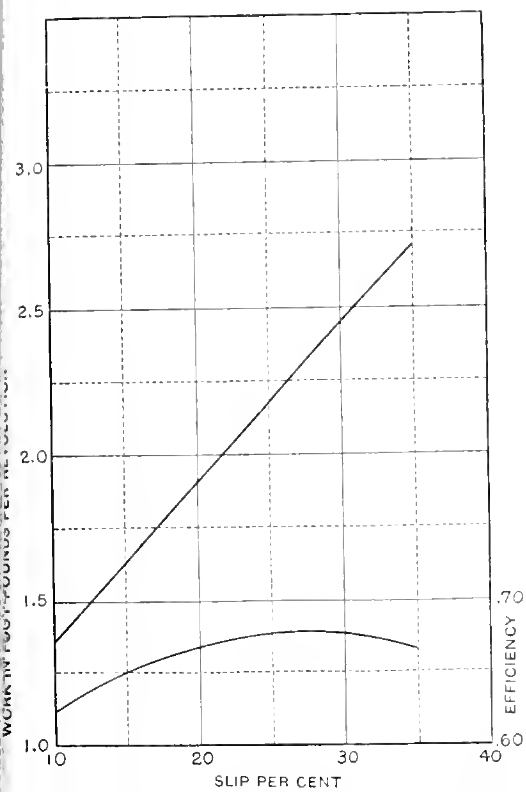

Fig. 49.—Tandem Propellers. Combination No. 29.

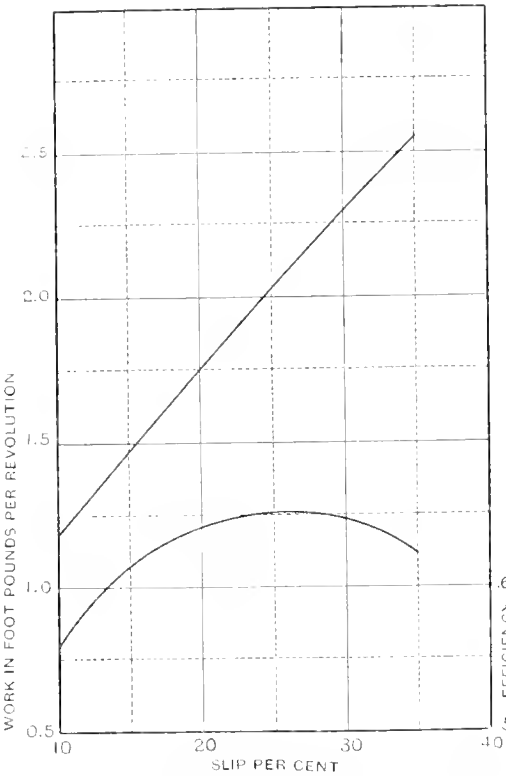

Fig. 50.—Tandem Propellers. Combination No. 30.

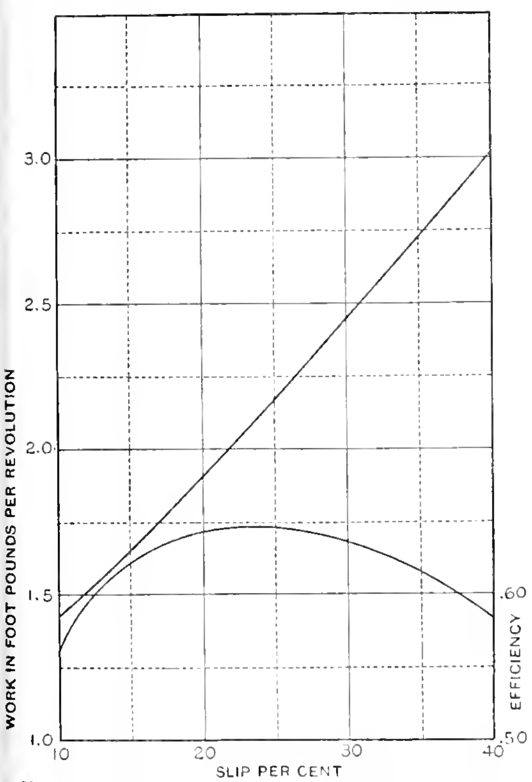

Fig. 51.—Tandem Propellers. Combination No. 31.

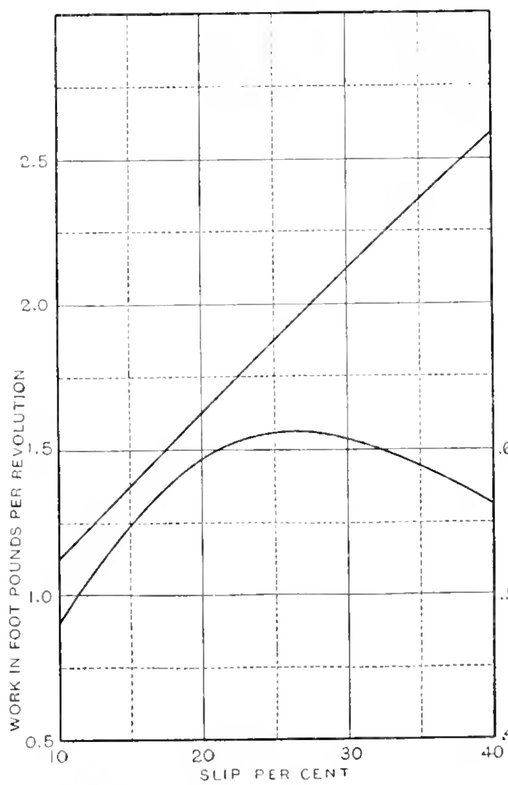

Fig. 52.—Tandem Propellers. Combination No. 32.

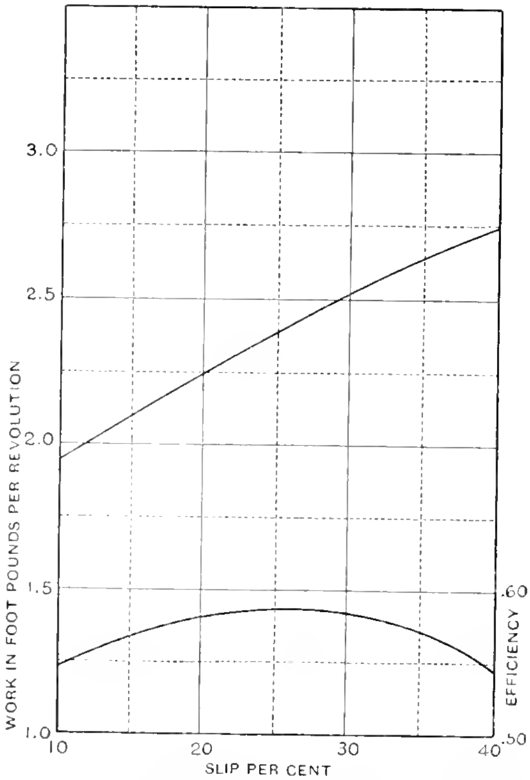

Fig. 53.—Tandem Propellers. Combination No. 33.

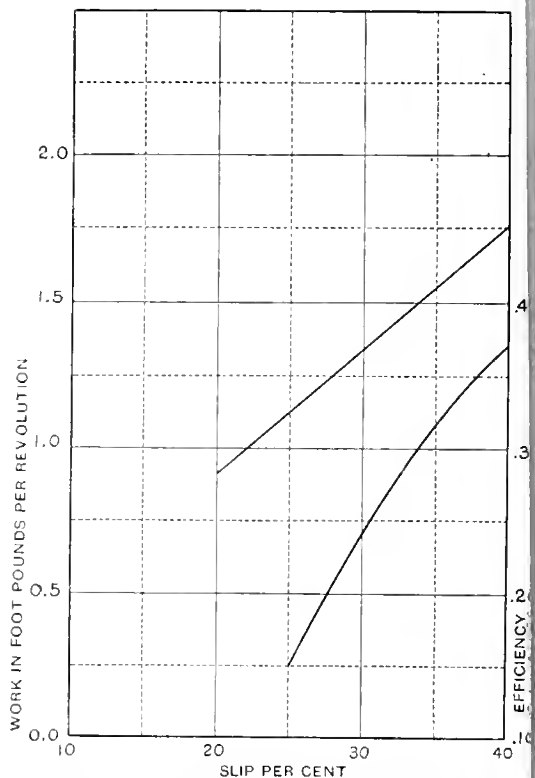

Fig. 54.—Tandem Propellers. Combination No. 3

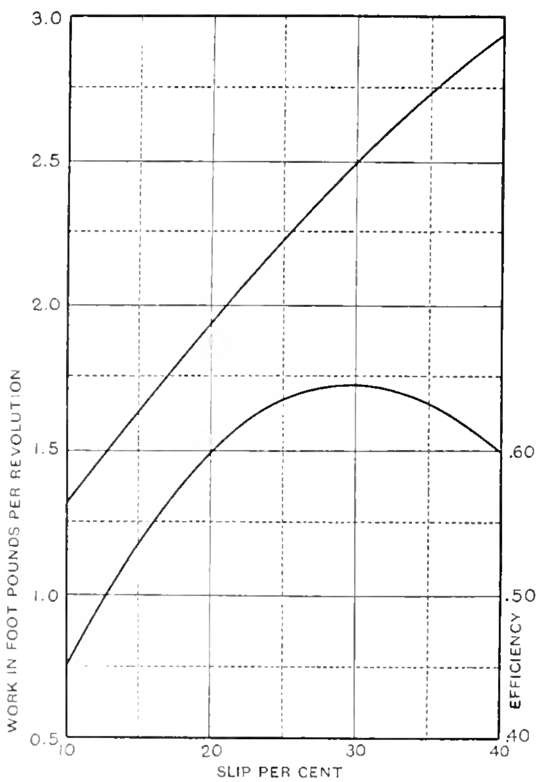

Fig. 55.—Tandem Propellers. Combination No. 35.

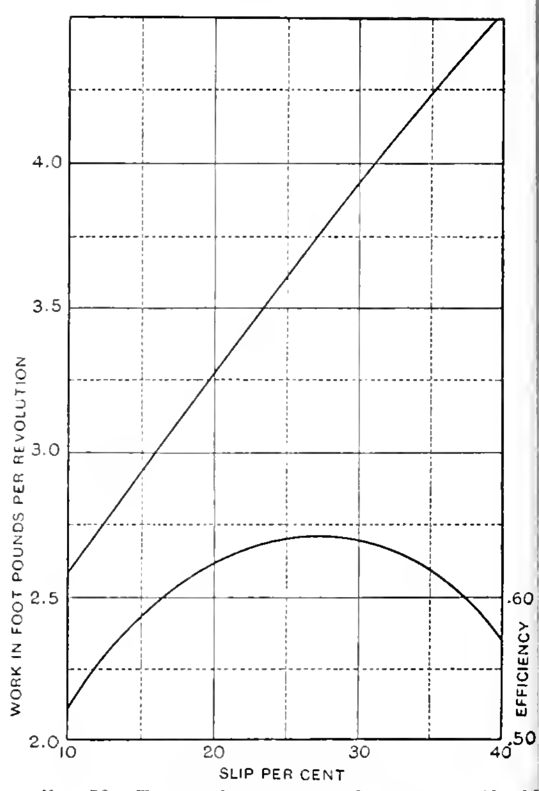

Fig. 56.—Tandem Propellers. Combination No. 36

As a further basis of comparison with the performances of the individual components of the pair, we may compare the work absorbed for the pair with that for the components under the same conditions of revolutions and speed of advance. To this end two illustrations will be sufficient, Nos. 8 and 19. In the former the components are of pitch 1.1 and 1.5 and the virtual pitch is 1.32, and hence for 100 revolutions per minute the product pN is 132. Then for 20 per cent slip, for example, the speed of advance will be $.80 \times 132 = 105.6$.

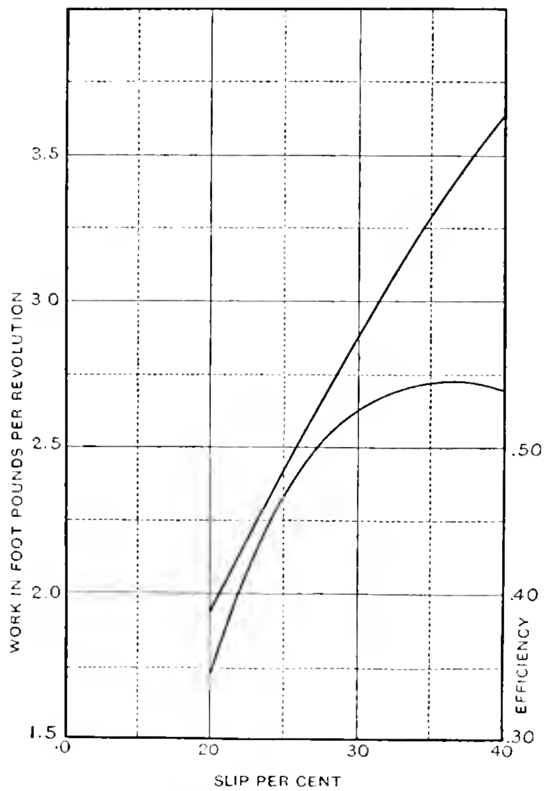

FIG. 57.—TANDEM PROPELLERS. COMBINATION NO. 37.

Now with 100 revolutions and this speed of advance the slip of a propeller of 1.1 pitch will be 4 per cent and for one of 1.5 it will be about 30 per cent. Then

W for 1.1-5 at 4 per cent slip	.60
W for 1.5-5 at 30 per cent slip	1.90
The sum	= 2.50
Actual W for combination	= 2.15

Again for No. 19 the two components are of pitch 1.5 and 1.7 and the virtual pitch is 1.59. Then we find, as above, that the 1.5 propeller operates at a slip of about 15 and the 1.7 at about 25 and

W for 1.5-5 at 15 per cent slip	1.30
W for 1.7-4 at 25 per cent slip	1.85
The sum	= 3.15
Actual W for combination	= 2.45

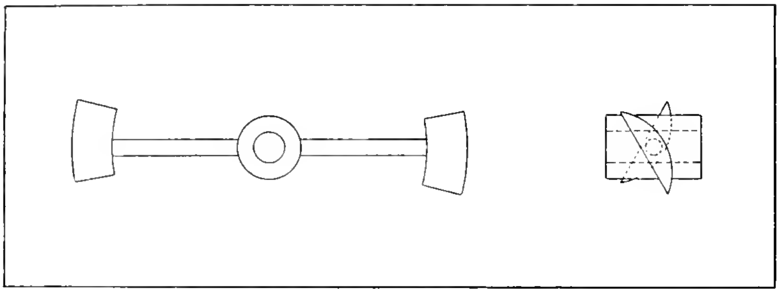

FIG. 58.—ELEMENT OF PROPELLER BLADE.

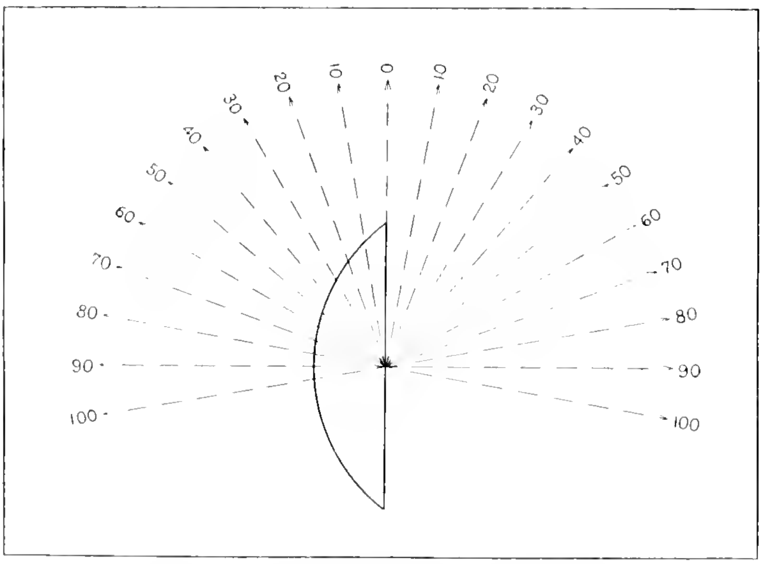

FIG. 59.—ANGLES OF APPROACH FOR BLADE ELEMENT.

INVESTIGATION OF ELEMENT OF PROPELLER BLADE.

In order to determine certain fundamental data with regard to the nature of the forces acting on a body having the form of an element of a propeller blade, a series of tests was made on a model as shown in fig. 58. The area of the face of the element was about 4 square inches and the outside diameter about 12 inches. By a suitable adjustment the angularity of the element with reference to the transverse plane could be changed at will and set at any

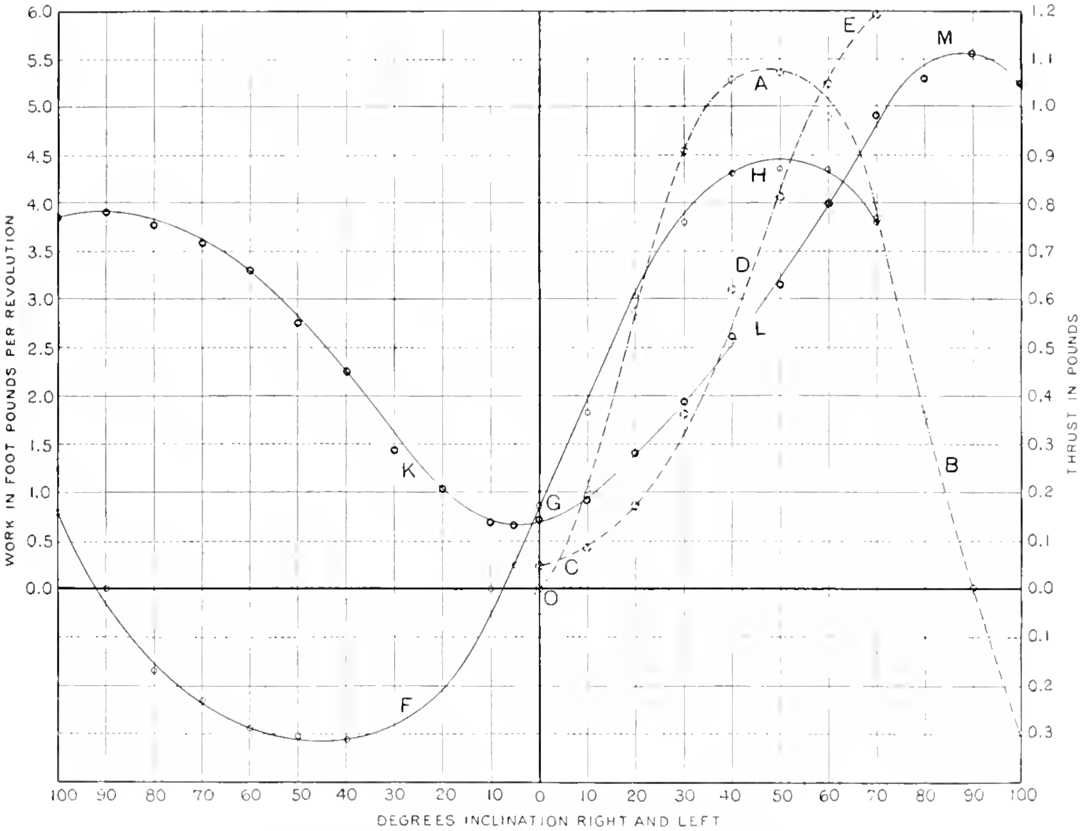

FIG. 60.—RESULTS FOR PLANE AND ROUNDED BACK ELEMENTS.

$O\ A\ B$ = Thrust for Plane Element
$C\ D\ E$ = Work in Foot Pounds per Revolution for Plane Element.
$F\ G\ H$ = Thrust for Rounded Back Element.
$K\ L\ M$ = Work in Foot Pounds per Revolution for Rounded Back Element.

desired point. The model was then run with the car stationary and with the element set at values of the inclination with the transverse plane regularly varying by 10-degree intervals. This gave a series of directions of element relative to water as shown by the arrows of fig. 59. For each of these settings runs were made at varying revolutions, all ultimately reduced to the standard of 100 revolutions per minute, and the resulting values of thrust and work in

foot-pounds per revolution are shown by the curves of fig. 60. The influence of the rods and of the hub on the work was eliminated by suitable experiments with these alone, so that the final values as shown in the diagram represent forces on the model element alone. It is readily seen that so far as motion relative to the water is concerned, the forces developed on the model itself when revolved with its plane face in a transverse plane, or at a zero angle of inclination, will be similar to those when operating at any angle of inclination combined with such speed of advance as to give the conditon for zero slip relative to the face. In other words, in each case the element moves in the direction of its plane face relative to the water. The remarkable result of motion in this direction is shown to be a definite force acting against the plane face as shown by the value of the thrust for zero angle of inclination in fig. 60. In the case of a propeller this will give a positive thrust for zero slip, a result actually observed in practically every case under observation. This seemingly anomalous result is due to the peculiar distribution of the stream lines about a body of the cross-section of a propeller blade, a subject which would repay in scientific interest some further examination. The same general result has been indicated by results of other experiments, and it is seemingly well established from the propellers themselves that with a plane face and rounded back a propeller will give a positive thrust for zero slip, the pitch being based on the inclination of the driving face. The results given by this model demonstrate the same fact and confirm in a clear and striking manner the indications derived from the propellers at zero slip. The existence of a forward thrust for zero angle with the transverse plane was very clearly indicated not only by the dynamometric apparatus itself, but also by the stream of water which was unmistakably sent aft by the operation of the model in this manner.

In order to compare with the results obtained from the element of blade with rounded back, a series of similar measurements were made with a model of the same general dimensions but of thin sheet steel with sharp edges and hence representing as nearly as possible a plane element. In this case, however, the examination was made for angles of inclination in one direction only, as due to symmetry the same results would necessarily be found for equal inclination in the other direction. These results are likewise shown in fig. 60 by OAB, CDE.

A comparison of the results for these two models is rich in suggestions regarding the influence of the stream lines about an irregular body of the form used. In particular it may be noted that for all angles up to about 20 degrees the thrust developed by the plane element is less than that given by the element with curved back while in the case of turning moment the same relation holds up to nearly 40 degrees. It thus appears that the influence of the curved back is such as to produce at moderate angles an increase in the forces acting on the plane face, and in particular an increase in the forward thrust and

the transverse turning moment as compared with the values for a plane element with the same face area.

It seems likely from these indications that the forces on the face of a propeller blade operating at any moderate value of the slip, and corresponding here with an angle of inclination with the transverse plane not perhaps exceeding 10 degrees, will, on the whole, slightly exceed those which would be developed by a plane blade under similar conditions, and that hence the net result of the influence of the rounded back of a propeller blade is to increase the thrust as compared with a blade without thickness and operating under like conditions. For low values of the slip the resistance to turning or the work per revolution is increased in still greater ratio and the efficiency is reduced as compared with that for a blade without thickness. For high values of the slip it is by no means sure that such is the case, and there are not lacking indications that due to the rounded back the value of the efficiency at high values of the slip may exceed those for a blade without thickness and operating under like conditions of slip. The complete explanation of these points is still far from being on a satisfactory basis, however, and they merit further examination with reference to a better understanding of the influence of these various factors on propeller efficiency.

APPENDIX I.

AREA NUMBER 2.

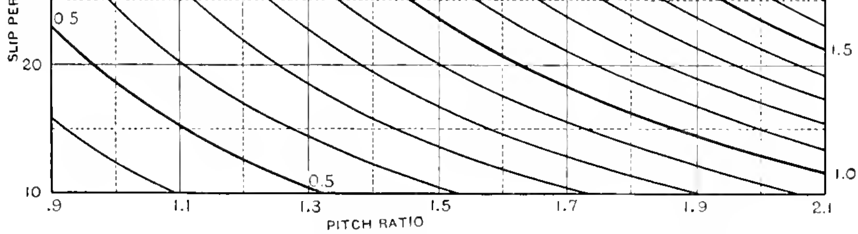

FIG. 61.—VALUES OF W ON PITCH-SLIP CONTOURS. CONTOURS MARKED IN FOOT POUND UNITS.

AREA NUMBER 3.

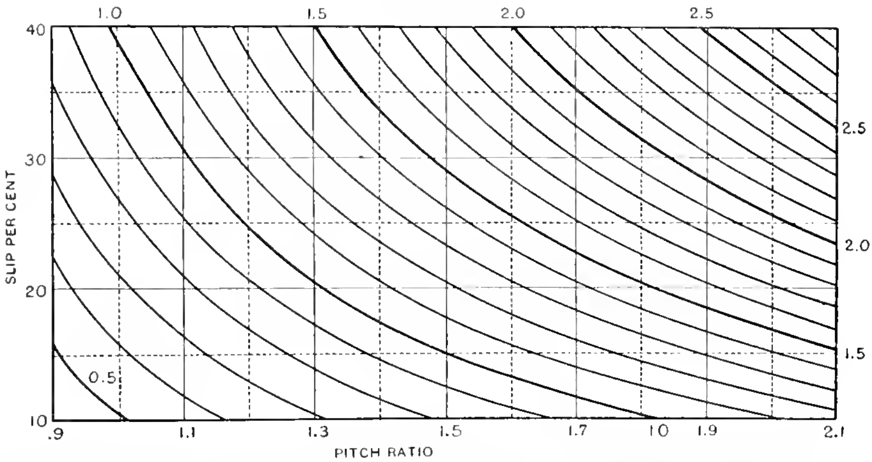

FIG. 62.—VALUES OF W ON PITCH-SLIP CONTOURS. CONTOURS MARKED IN FOOT POUND UNITS.

AREA NUMBER 4.

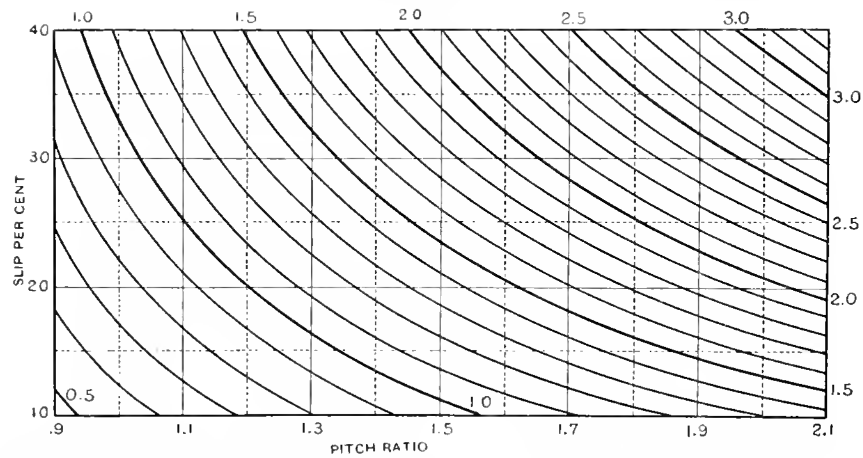

FIG. 63.—VALUES OF W ON PITCH-SLIP CONTOURS. CONTOURS MARKED IN FOOT POUND UNITS.

AREA NUMBER 5.

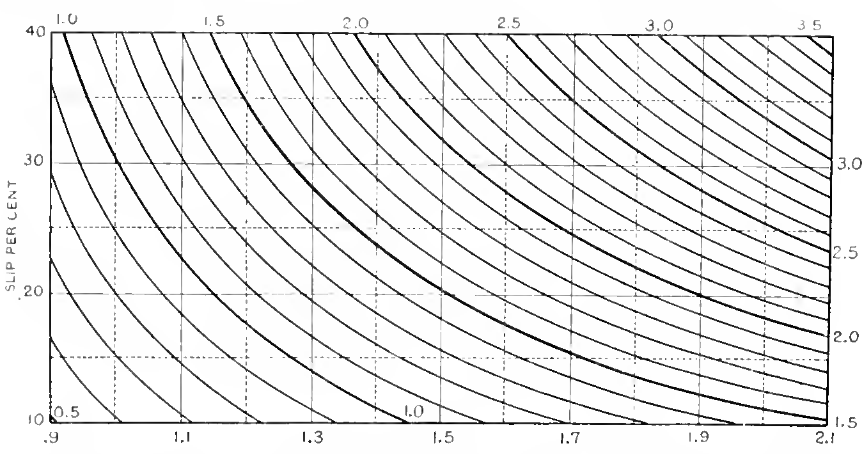

FIG. 64.—VALUES OF W ON PITCH-SLIP CONTOURS. CONTOURS MARKED IN FOOT POUND UNITS.

AREA NUMBER 6.

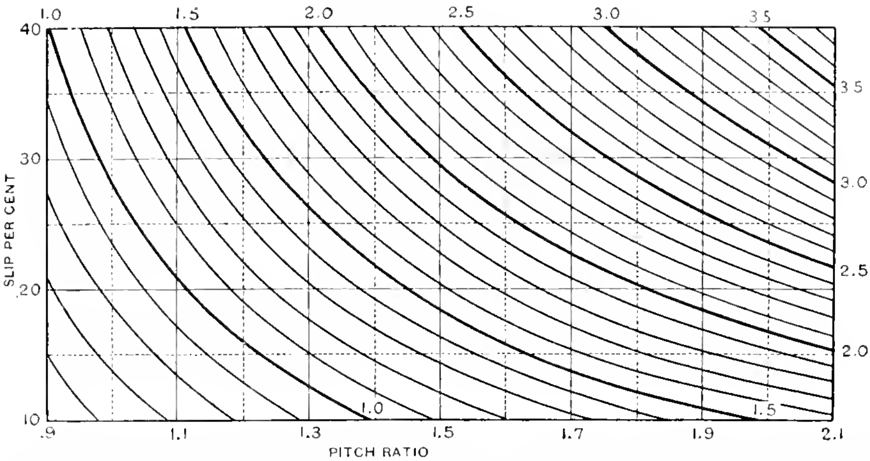

FIG. 65.—VALUES OF W ON PITCH-SLIP CONTOURS. CONTOURS MARKED IN FOOT POUND UNITS.

AREA NUMBER 7.

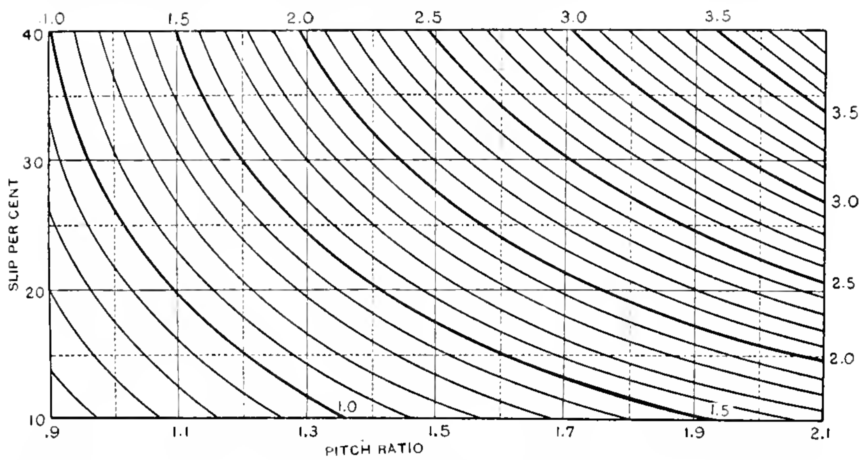

FIG. 66.—VALUES OF W ON PITCH-SLIP CONTOURS. CONTOURS MARKED IN FOOT POUND UNITS.

AREA NUMBER 8.

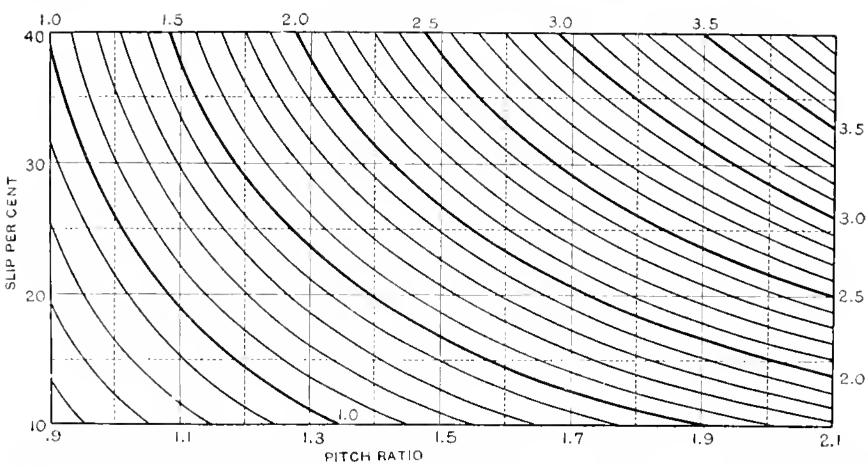

FIG. 67.—VALUES OF W ON PITCH-SLIP CONTOURS. CONTOURS MARKED IN FOOT POUND UNITS.

Pitch-Ratio 0.9.

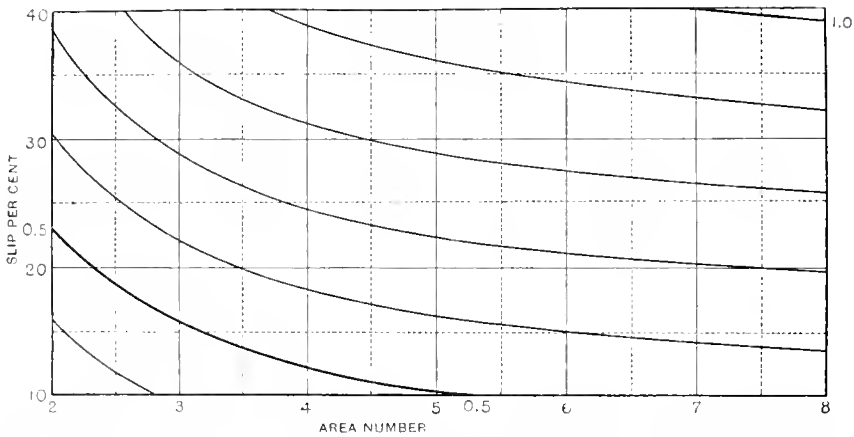

Fig. 68.—Values of W on Area-Slip Contours. Contours Marked in Foot Pound Units.

Pitch-Ratio 1.1.

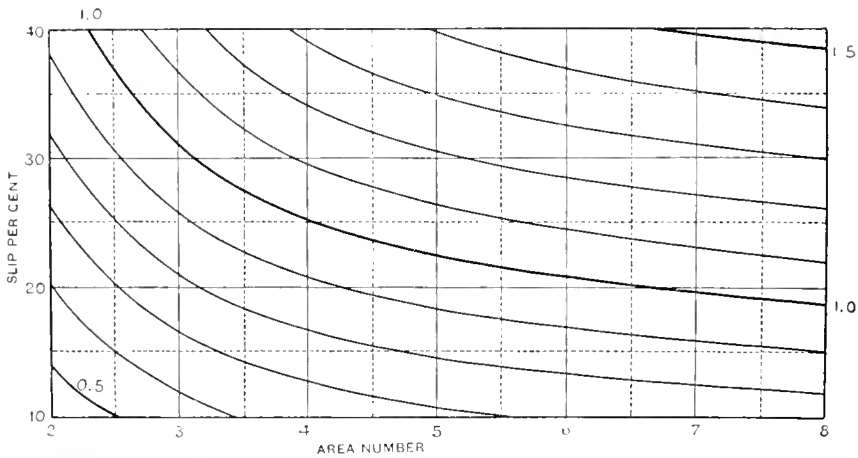

Fig. 69.—Values of W on Area-Slip Contours. Contours Marked in Foot Pound Units.

Pitch-Ratio 1.3.

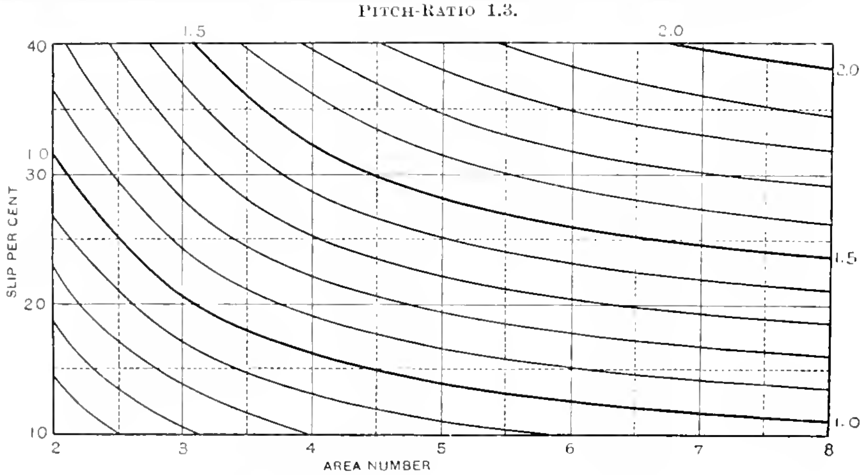

Fig. 70.—Values of W on Area-Slip Contours. Contours Marked in Foot Pound Units.

Pitch-Ratio 1.5.

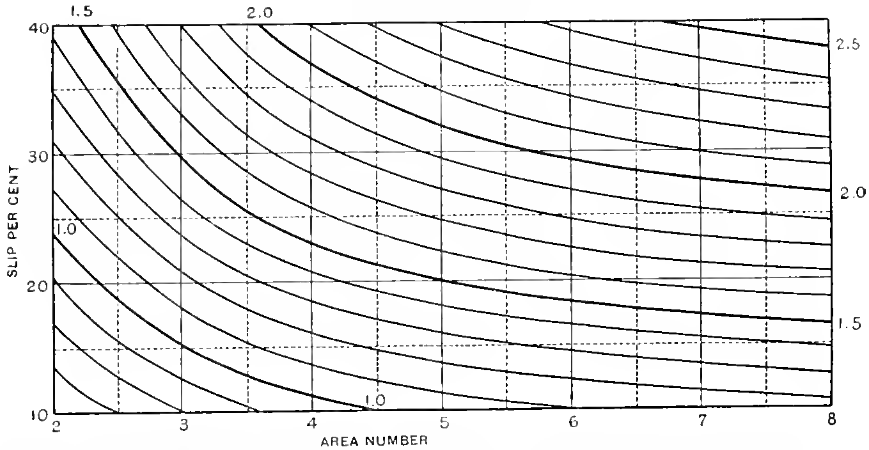

Fig. 71.—Values of W on Area-Slip Contours. Contours Marked in Foot Pound Units.

Pitch-Ratio 1.7.

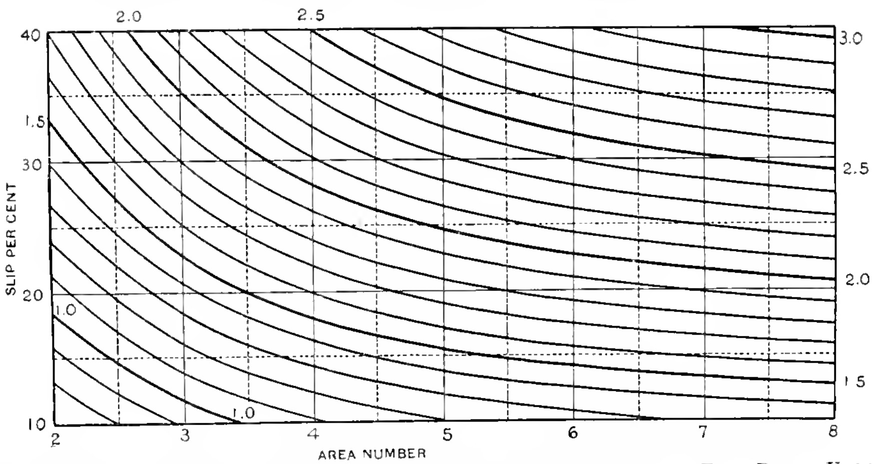

Fig. 72.—Values of W on Area-Slip Contours. Contours Marked in Foot Pound Units.

Pitch-Ratio 1.9.

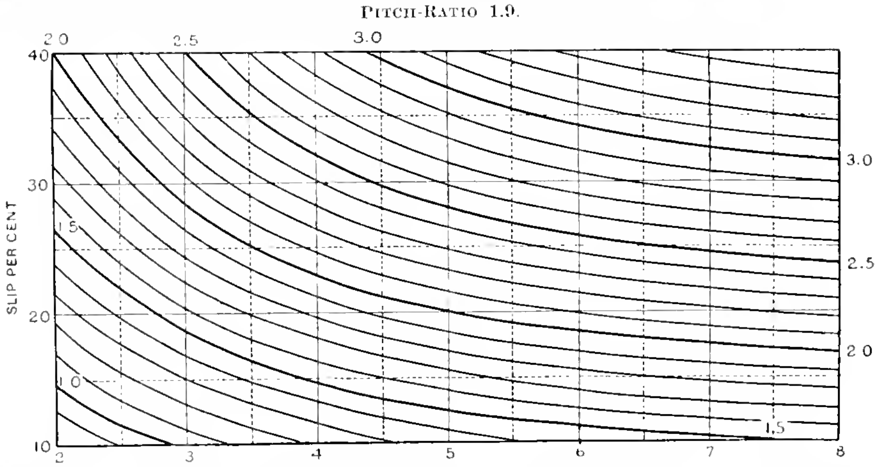

Fig. 73.—Values of W on Area-Slip Contours. Contours Marked in Foot Pound Units.

PITCH-RATIO 2.1.

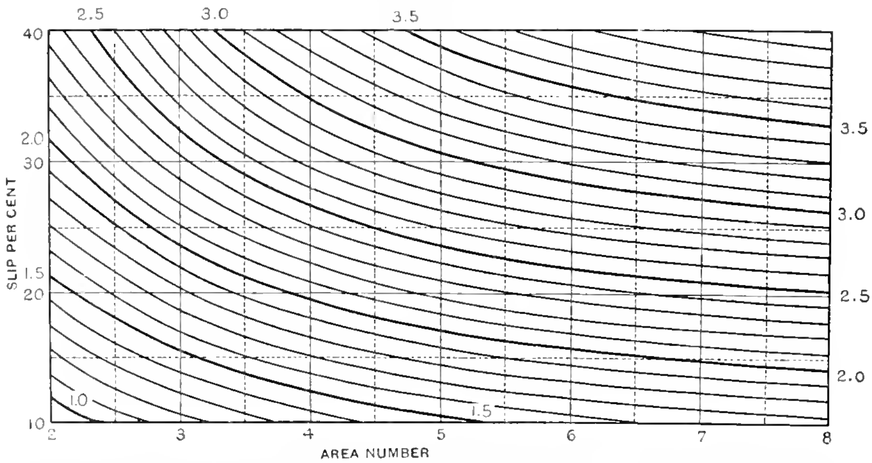

FIG. 74.—VALUES OF W ON AREA-SLIP CONTOURS. CONTOURS MARKED IN FOOT POUND UNITS.

SLIP 10 PER CENT.

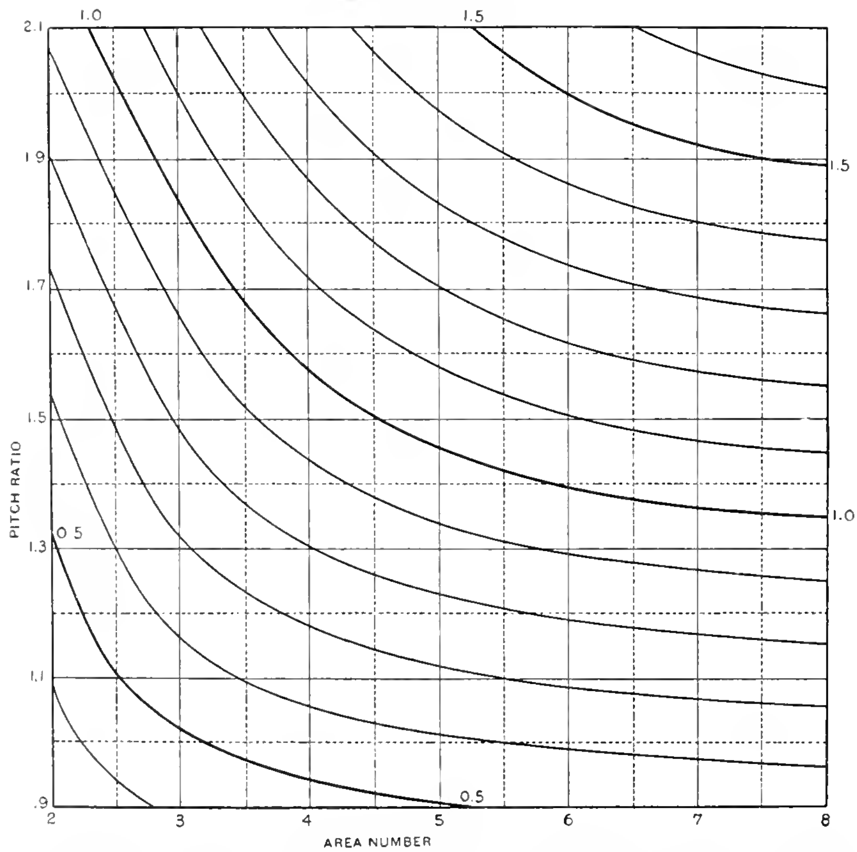

FIG. 75.—VALUES OF W ON PITCH-AREA CONTOURS. CONTOURS MARKED IN FOOT POUND UNITS.

SLIP 20 PER CENT.

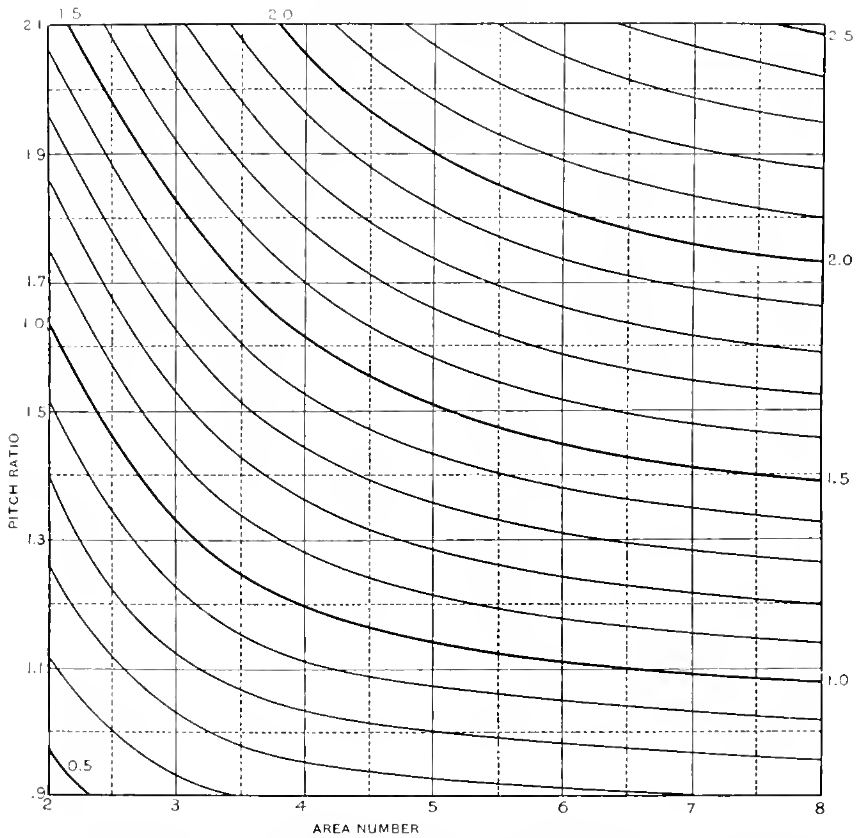

FIG. 76.—VALUES OF W ON PITCH-AREA CONTOURS. CONTOURS MARKED IN FOOT POUND UNITS.

SLIP 30 PER CENT.

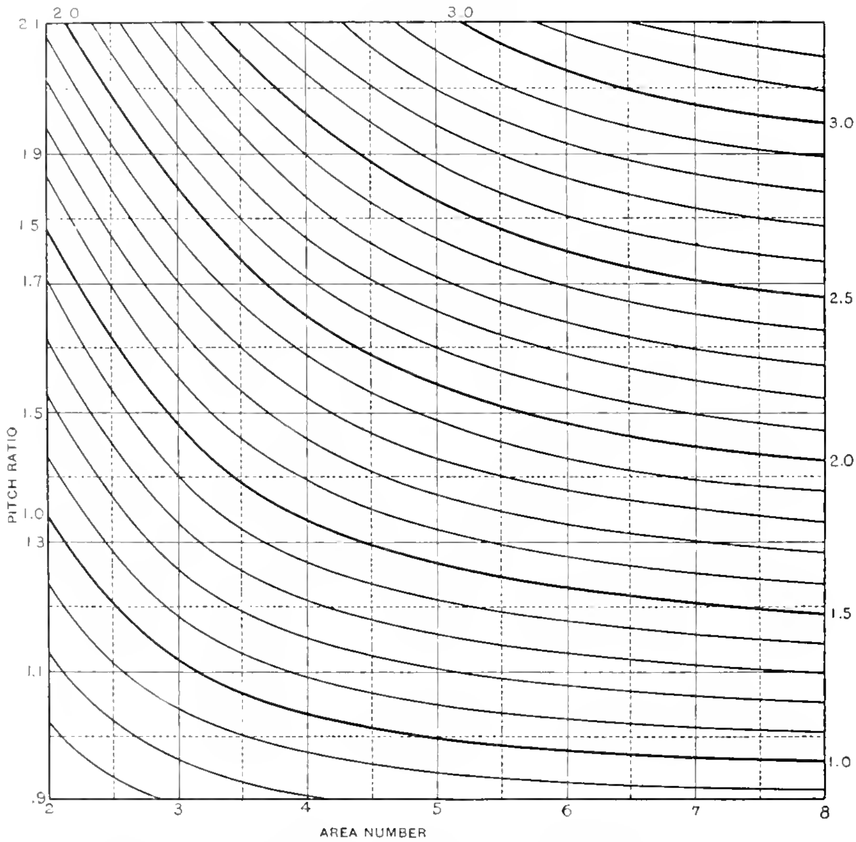

FIG. 77.—VALUES OF W ON PITCH-AREA CONTOURS. CONTOURS MARKED IN FOOT POUND UNITS.

SLIP 40 PER CENT.

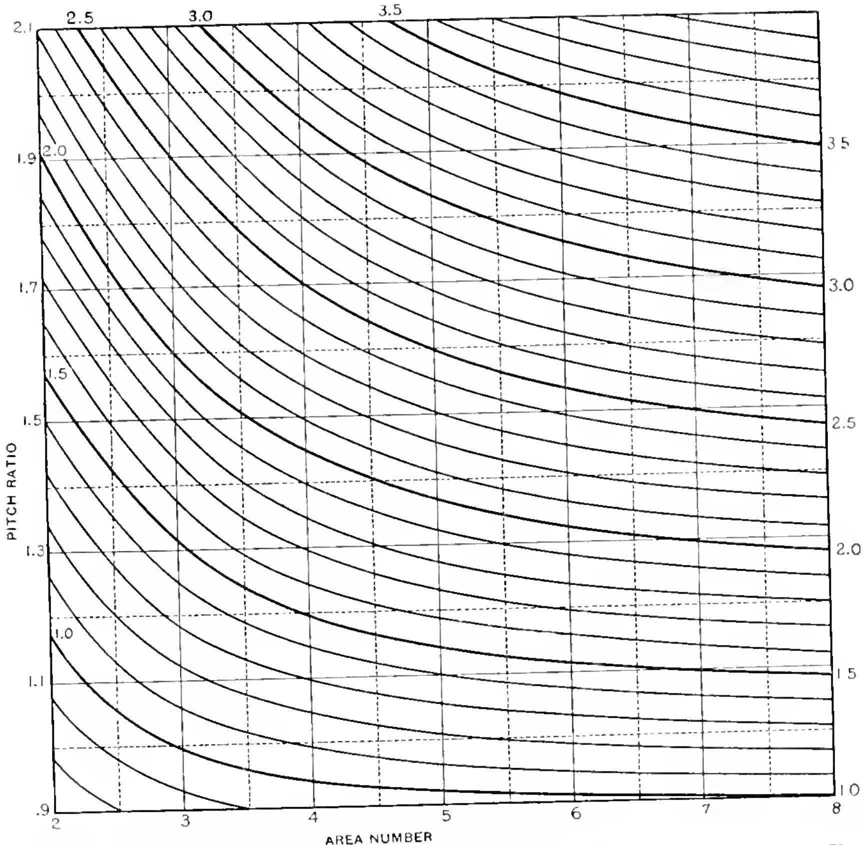

FIG. 78.—VALUES OF W ON PITCH-AREA CONTOURS. CONTOURS MARKED IN FOOT POUND UNITS.

AREA NUMBER 2.

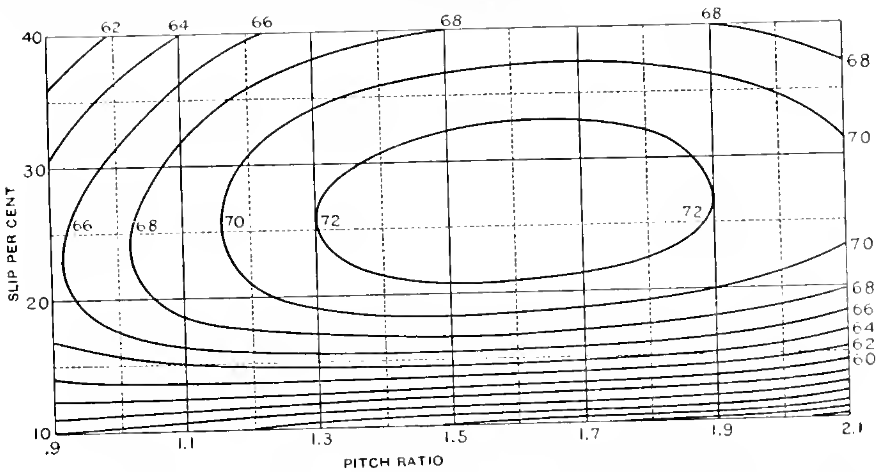

FIG. 79.—VALUES OF EFFICIENCY ON PITCH-SLIP CONTOURS. CONTOURS MARKED IN PER CENT EFFICIENCY.

Area Number 3.

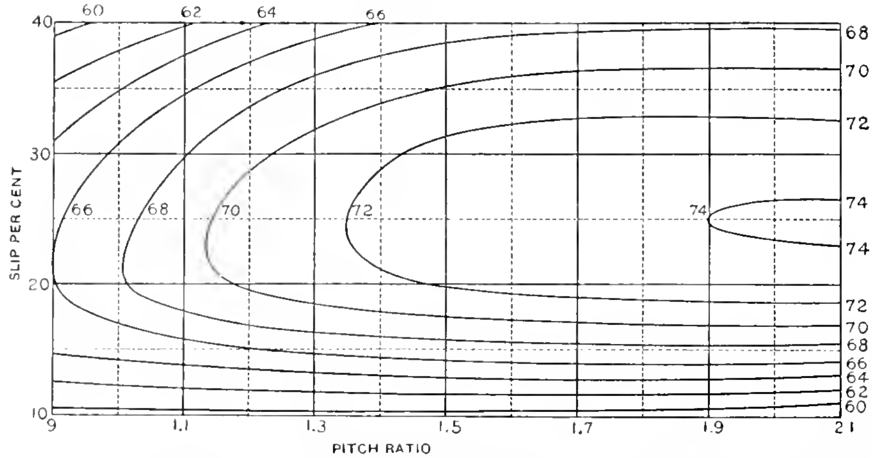

Fig. 80.—Values of Efficiency on Pitch-Slip Contours. Contours Marked in Per Cent Efficiency.

Area Number 4.

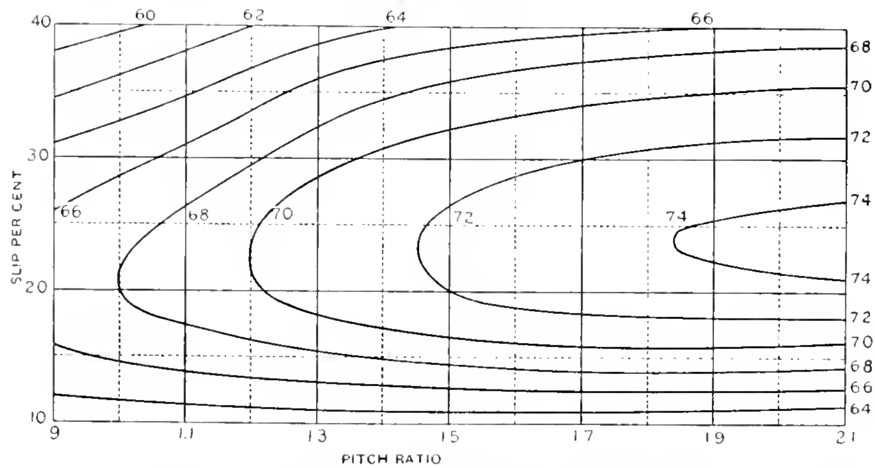

Fig. 81.—Values of Efficiency on Pitch-Slip Contours. Contours Marked in Per Cent Efficiency.

Area Number 5.

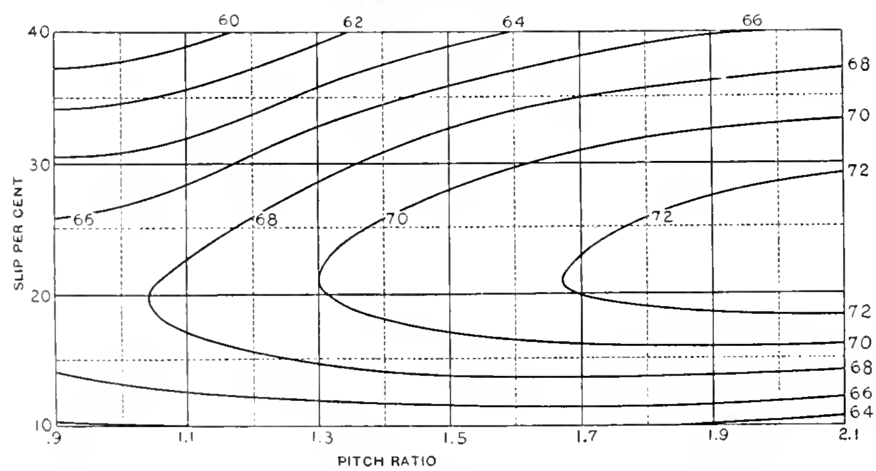

Fig. 82.—Values of Efficiency on Pitch-Slip Contours. Contours Marked in Per Cent Efficiency.

Area Number 6.

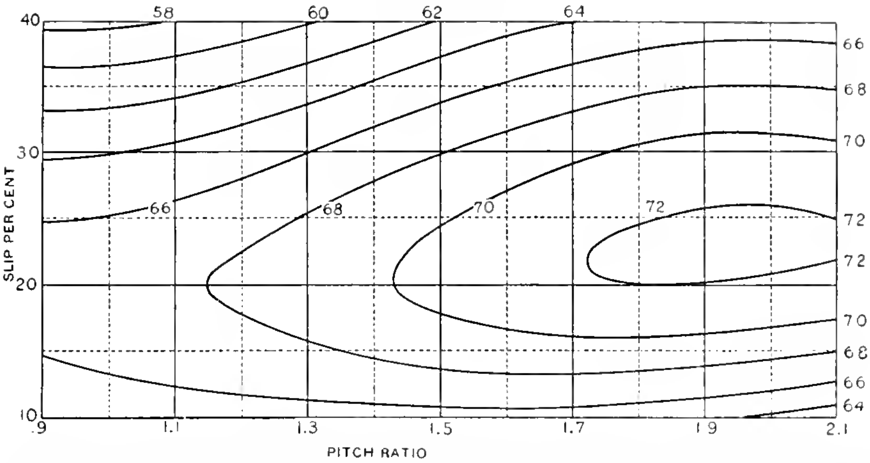

Fig. 83.—Values of Efficiency on Pitch-Slip Contours. Contours Marked in Per Cent Efficiency.

Area Number 7.

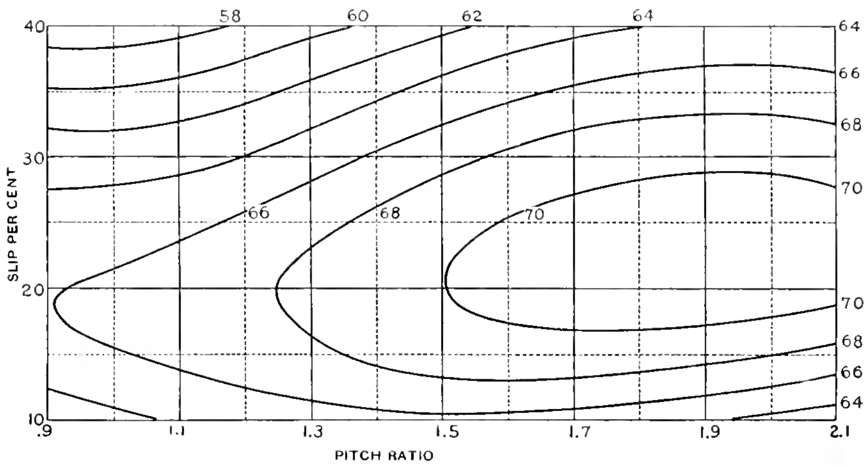

Fig. 84.—Values of Efficiency on Pitch-Slip Contours. Contours Marked in Per Cent Efficiency.

Area Number 8.

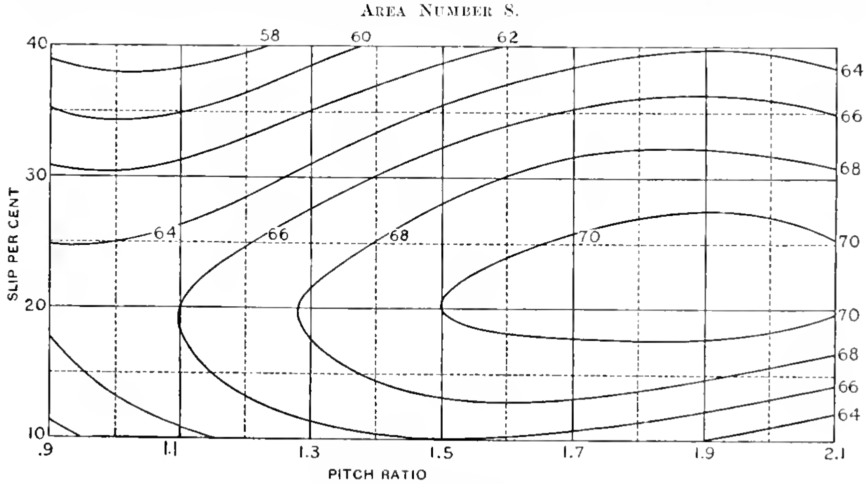

Fig. 85.—Values of Efficiency on Pitch-Slip Contours. Contours Marked in Per Cent Efficiency.

APPENDIX II.

TABLE 4.

Pitch ratio and Area No.	Slip per cent.	Thrust in pounds.	Work in foot pounds per revolution.	Efficiency.
Pitch ratio .9..... Area number 2.....	10	.225	.325	.561
	20	.419	.463	.652
	30	608	.598	.640
	40	.800	.720	.600
Pitch ratio .9..... Area number 3.....	10	.310	.420	.598
	20	.525	.573	.660
	30	.735	.720	.643
	40	.943	.858	.594
Pitch ratio .9..... Area number 4.....	10	.363	.470	.625
	20	.588	.633	.669
	30	.803	.785	.644
	40	1.008	.925	.588
Pitch ratio .9..... Area number 5.....	10	.390	.495	.638
	20	.616	.660	.672
	30	.838	.820	.644
	40	1.043	.960	.582
Pitch ratio .9..... Area number 6.....	10	.405	.513	.640
	20	.635	.683	.670
	30	.855	.843	.639
	40	1.050	.983	.577
Pitch ratio .9..... Area number 7.....	10	.410	.528	.630
	20	.640	.700	.658
	30	.863	.860	.632
	40	1.058	.998	.573
Pitch ratio .9..... Area number 8.....	10	.408	.538	.614
	20	.638	.710	.646
	30	.863	.873	.623
	40	1.063	1.008	.570
Pitch ratio 1.1..... Area number 2.....	10	.225	.408	.547
	20	.468	598	.688
	30	.690	.775	.686
	40	.900	.928	.640
Pitch ratio 1.1..... Area number 3.....	10	.338	.563	.594
	20	.613	.783	.689
	30	.875	.993	.679
	40	1.110	1.175	.624
Pitch ratio 1.1..... Area number 4.....	10	.405	.640	.627
	20	.693	.885	.689
	30	.965	1.119	.664
	40	1.213	1.318	.607
Pitch ratio 1.1..... Area number 5.....	10	.440	.680	.641
	20	.731	.940	.685
	30	1.005	1.188	.652
	40	1.260	1.403	.593
Pitch ratio 1.1..... Area number 6.....	10	.463	.708	.647
	20	.755	.980	.678
	30	1.033	1.240	.641
	40	1.288	1.463	.581

TABLE 4.—Continued.

Pitch ratio and Area No.	Slip per cent.	Thrust in pounds.	Work in foot-pounds per revolution.	Efficiency.
Pitch ratio 1.1..... Area number 7.....	10	.475	.733	.642
	20	.770	1.013	.669
	30	1.050	1.280	.632
	40	1.308	1.505	.573
Pitch ratio 1.1..... Area number 8.....	10	.480	.750	.634
	20	.778	1.038	.660
	30	1.060	1.310	.623
	40	1.323	1.538	.568
Pitch ratio 1.3..... Area number 2.....	10	.225	.495	.532
	20	.500	.738	.705
	30	.760	.970	.715
	40	.995	1.163	.668
Pitch ratio 1.3..... Area number 3.....	10	.350	.693	.591
	20	.673	.985	.709
	30	.980	1.263	.707
	40	1.251	1.498	.652
Pitch ratio 1.3..... Area number 4.....	10	.430	.800	.629
	20	.770	1.128	.710
	30	1.098	1.443	.690
	40	1.388	1.713	.632
Pitch ratio 1.3..... Area number 5.....	10	.480	.873	.644
	20	.825	1.228	.699
	30	1.155	1.563	.673
	40	1.455	1.853	.613
Pitch ratio 1.3..... Area number 6.....	10	.503	.905	.650
	20	.853	1.283	.691
	30	1.193	1.643	.661
	40	1.500	1.950	.600
Pitch ratio 1.3..... Area number 7.....	10	.522	.938	.651
	20	.878	1.330	.686
	30	1.218	1.703	.651
	40	1.540	2.02	.595
Pitch ratio 1.3..... Area number 8.....	10	.530	.953	.651
	20	.890	1.358	.682
	30	1.238	1.740	.647
	40	1.568	2.068	.591
Pitch ratio 1.5..... Area number 2.....	10	.225	.588	.517
	20	.535	.893	.717
	30	.818	1.183	.726
	40	1.075	1.420	.681
Pitch ratio 1.5..... Area number 3.....	10	.353	.808	.589
	20	.708	1.180	.720
	30	1.055	1.530	.724
	40	1.355	1.825	.668
Pitch ratio 1.5..... Area number 4.....	10	.443	950	.628
	20	.820	1.368	.720
	30	1.193	1.765	.710
	40	1.520	2.103	.651

TABLE 4.—Continued.

Pitch ratio and Area No.	Slip per cent.	Thrust in pounds.	Work in foot-pounds per revolution.	Efficiency.
Pitch ratio 1.5..... Area number 5.....	10	.495	1.038	.644
	20	.885	1.493	.712
	30	1.268	1.925	.691
	40	1.615	2.295	.633
Pitch ratio 1.5..... Area number 6.....	10	.533	1.098	.655
	20	.928	1.578	.705
	30	1.318	2.035	.680
	40	1.678	2.425	.623
Pitch ratio 1.5..... Area number 7.....	10	.550	1.128	.659
	20	.955	1.633	.700
	30	1.355	2.113	.673
	40	1.725	2.523	.615
Pitch ratio 1.5..... Area number 8.....	10	.563	1.150	.660
	20	.973	1.668	.700
	30	1.380	2.160	.671
	40	1.753	2.578	.612
Pitch ratio 1.7..... Area number 2.....	10	.225	.683	.504
	20	.553	1.055	.712
	30	.858	1.408	.725
	40	1.140	1.693	.687
Pitch ratio 1.7..... Area number 3.....	10	.355	.925	.587
	20	.738	1.378	.728
	30	1.113	1.813	.730
	40	1.438	2.170	.676
Pitch ratio 1.7..... Area number 4.....	10	.446	1.090	.626
	20	.858	1.600	.729
	30	1.263	2.088	.720
	40	1.623	2.500	.662
Pitch ratio 1.7..... Area number 5.....	10	.505	1.200	.644
	20	929	1.753	.721
	30	1.350	2.280	.705
	40	1.738	2.730	.649
Pitch ratio 1.7..... Area number 6.....	10	.543	1.275	.651
	20	.976	1.858	.715
	30	1.405	2.413	.695
	40	1.805	2.888	.638
Pitch ratio 1.7..... Area number 7.....	10	.560	1.313	.653
	20	1.004	1.923	.710
	30	1.448	2.500	.689
	40	1.850	2.993	.631
Pitch ratio 1.7..... Area number 8.....	10	.570	1.338	.652
	20	1.015	1.958	.705
	30	1.473	2.550	.687
	40	1.885	3.055	.629
Pitch ratio 1.9..... Area number 2.....	10	.228	.795	489
	20	.570	1.238	.700
	30	893	1.655	.717
	40	1.185	1.988	.680

TABLE 4.—Continued.

Pitch ratio and Area No.	Slip per cent.	Thrust in pounds.	Work in foot-pounds per revolution.	Efficiency.
Pitch ratio 1.9..... Area number 3.....	10	.358	1.045	.585
	20	.763	1.583	.732
	30	1.150	2.093	.731
	40	1.493	2.513	.677
Pitch ratio 1.9..... Area number 4.....	10	.448	1.225	.624
	20	.883	1.830	.733
	30	1.310	2.403	.725
	40	1.698	2.888	.670
Pitch ratio 1.9..... Area number 5.....	10	.505	1.350	.640
	20	.956	2.003	.720
	30	1.405	2.625	.712
	40	1.825	3.153	.660
Pitch ratio 1.9..... Area number 6.....	10	.540	1.438	.643
	20	1.003	2.118	.719
	30	1.470	2.765	.707
	40	1.895	3.315	.652
Pitch ratio 1.9..... Area number 7.....	10	.555	1.478	.642
	20	1.025	2.185	.713
	30	1.495	2.858	.696
	40	1.935	3.430	.643
Pitch ratio 1.9..... Area number 8.....	10	.566	1.513	.640
	20	1.040	2.230	.709
	30	1.513	2.918	.690
	40	1.963	3.500	.639
Pitch ratio 2.1..... Area number 2.....	10	.230	.915	.475
	20	.585	1.438	.684
	30	.923	1.930	.703
	40	1.228	2.325	.665
Pitch ratio 2.1..... Area number 3.....	10	.360	1.168	.583
	20	.783	1.795	.732
	30	1.188	2.388	.731
	40	1.545	2.880	.676
Pitch ratio 2.1..... Area number 4.....	10	.448	1.363	.620
	20	.903	2.060	.736
	30	1.350	2.725	.728
	40	1.748	3.288	.670
Pitch ratio 2.1..... Area number 5.....	10	.495	1.485	.631
	20	.974	2.240	.730
	30	1.445	2.963	.717
	40	1.875	3.578	.660
Pitch ratio 2.1..... Area number 6.....	10	.525	1.573	.631
	20	1.010	2.373	.715
	30	1.500	3.133	.703
	40	1.950	3.785	.649
Pitch ratio 2.1..... Area number 7.....	10	.543	1.628	.630
	20	1.033	2.455	.707
	30	1.525	3.243	.691
	40	1.990	3.918	.640
Pitch ratio 2.1..... Area number 8.....	10	.550	1.675	.621
	20	1.050	2.513	.702
	30	1.543	3.315	.684
	40	2.008	4.000	.632